[This question paper contains 4 printed pages.]

**Sr. No. of Question Paper : 1846** **GC-3** **Your Roll No...............**

Unique Paper Code : 42354302

Name of the Paper : Paper III - Algebra

Name of the Course : **B.Sc. Physical Sciences / Mathematical Sciences (Part – II)**

Semester : III (Under CBCS)

Duration : 3 Hours Maximum Marks : 75

**<u>Instructions for Candidates</u>**

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Marks are indicated.

**UNIT – I**

1. (a) Define Group. Give examples of each of following :

   (i) Finite abelian group.

   (ii) Finite non-abelian group.

   (iii) Infinite abelian group.

   (iv) Infinite non-abelian group.

   (v) Cyclic group.

   (vi) Abelian group which is not cyclic. (6)

(b) Let G be a group. Show that $Z(G) = \bigcap_{a \in G} C(a)$. Where Z(G) is the center of the group G and C(a) is the centralizer of a in G. (6)

(c) Find the inverse of the element $\begin{bmatrix} 2 & 6 \\ 3 & 5 \end{bmatrix}$ in $GL(2, Z_{11})$. (6)

2. (a) Let H be a nonempty finite subset of a group G. Then show that if is a subgroup of G if H is closed under the operation of G. (6)

(b) Define cyclic group. Give an example of a noncyclic group, all of whose proper subgroups are cyclic. (6)

(c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix}$ and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}$

Compute each of the following :

(a) $\alpha^{-1}$

(b) $\beta\alpha$

(c) $\alpha\beta$ (6)

3. (a) State and prove Lagrange's theorem for finite group. (6)

(b) Find all left cosets of $\{1,11\}$ in U(30). (6)

(c) Show that the order of a permutation on a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (6)

## UNIT – II

4. (a) Prove that a nonempty subset S of a ring R is a subring of R if and only if

   (i) $a - b \in S$ and (ii) $ab \in S$ for all $a,b \in S$.

   Hence show that if a is a fixed element of a ring R then $I_a = \{x \in R : ax = 0\}$ is a subring of R. (6½)

   (b) Let R be a commutative ring. Then show that R is an integral domain if and only if $ab = ac \Rightarrow b = c$, where $a,b,c \in R$ and $a \neq 0$. (6½)

   (c) Define an ideal of a ring R and prove that intersection of two ideals of a ring is an ideal but union is not so. (6½)

## UNIT – III

5. (a) Prove that a nonempty subset W of a vector space V(F) is a subspace of V if and only if $\alpha w_1 + \beta w_2 \in W \quad \forall \alpha, \beta \in F$ and $w_1, w_2 \in W$.

   Give an example of a nonempty subset W of a vector space V(F) which is not a subspace of V. (6½)

   (b) Show that the vectors (1,2,3,4),(0,1,–1,2),(1,5,1,8),(3,7,8,14) in $\mathbb{R}^4$ are linearly dependent over $\mathbb{R}$. (6½)

   (c) Determine whether or not the vectors (1,–3,2), (2,4,1) and (1,1,1) form a basis of $\mathbb{R}^3$. (6½)

6. (a) Define a basis of a vector space over a field F and prove that every element of a vector space is uniquely expressible as a linear combination f elements of the basis. (6½)

(b) (i) Show that the mapping $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$T(x_1,x_2) = (x_1 + x_2, x_1 - x_2, x_2)$ is a linear transformation.

(ii) Let $T : V \to W$ be a linear transformation. Prove that the vectors $v_1, v_2, v_3 \in V$ are linearly independent, if $T(v_1), T(v_2), T(v_3)$ are linearly independent. (6½)

(c) Let $T : V \to U$ be a linear transformation. Define null space $N(T)$ and range space $R(T)$ of T. Show that $N(T)$ is a subspace of V and $R(T)$ is a subspace of U. (6½)