[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1844 **GC-3** **Your Roll No................**

Unique Paper Code : 42374301

Name of the Paper : Statistical Inference

Name of the Course : **B.Sc. Mathematical Science STATISTICS (CBCS)**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt **Six** questions in all, selecting **two** questions from **Section–I** and **four** questions from **Section–II.**
3. Simple calculator is allowed.

Section – I

1. (a) Discuss in detail, the test of significance for difference of means for large samples.

(b) The demand for a particular spare part in a factory was found to vary from day-to-day. In a sample study the following information was obtained :

Day :	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.
No of parts demanded :	1124	1125	1110	1120	1126	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week. (Given that the values of chi-square significance at 5, 6, 7, d.f. are respectively 11.07, 12.59, 14.07 at the 5% level of significance.) (6½,6)

2. (a) If X_1 and X_2 are two independent chi-square variates with n_1 and n_2 d.f. respectively, then find the distribution of $X_1/(X_1 + X_2)$.

(b) State and prove the conditions under which

(i) Chi-square distribution reduces to negative exponential distribution

(ii) t-distribution reduces to cauchy distribution ? (6½,6)

3. (a) Define fisher's t-statistic. Derive the p.d.f. of fisher's 't' statistic. Show that student's 't' is a particular case of fisher's 't'.

(b) Show that the additive property holds good in the case of chi-square distribution. (8½,4)

Section-II

4. (a) What is meant by a statistical hypothesis ? Explain the concept of type I and type II errors with examples.

(b) Define sampling distribution and standard error of a statistic. Obtain standard error of mean when population is large. (6½,6)

5. (a) Explain the concept of Consistency. If $X_1, X_2, \ldots X_n$ are random observations on a Bernoulli variate X taking the value 1 with probability p and the value 0 with probability $(1 - p)$ then show that :

$$\frac{\sum x_i}{n}\left(1-\frac{\sum x_i}{n}\right)$$ is a consistent estimator of $p(1 - p)$.

(b) A random sample (X_1, X_2, X_3) of size 3 is drawn from a normal population with mean μ and variance σ^2. T_1, T_2, T_3 are the estimators used to estimate mean value μ, where

$$T_1 = X_1 + X_2 - X_3,\ T_2 = 2X_1 + 3X_3 - 4X_2 \text{ and } T_3 = \frac{(\lambda X_1 + X_2 + X_3)}{3}$$

(i) Are T_1 and T_2 unbiased estimators ?

(ii) Find the value of λ such that T_3 is unbiased estimator for μ.

(iii) Which is the best estimator ? (6½,6)

6. (a) Prove that an M.V.U is unique in the sense that if T_1 and T_2 are M.V.U. estimators for $\gamma(\theta)$, then $T_1 = T_2$, almost everywhere.

(b) Let $x_1, x_2, \ldots\ldots, x_n$ be a random sample from normal population $N(\mu, \sigma^2)$. Find sufficient estimators for μ and σ^2. (6½,6)

7. (a) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$, on the basis of single observation from the population

$$f(x, \theta) = \theta \exp(-\theta x); \quad 0 \leq x < \infty$$

Obtain the values of type I and type II errors.

(b) Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also find its variance. (6,6½)

8. Write short notes on any **two** of the following :

   (i) Relationship between t, F and chi- square distribution.

   (ii) Maximum likelihood method (with illustration) and its properties.

   (iii) Confidence interval for the parameters of normal distribution. (12½)