[This question paper contains 4 printed pages.]

4621 *Your Roll No.* .....

B.Sc. Prog./III **AS**

MA 302 – MATHEMATICS

Analysis, Algebra and Mechanics

(For Physical Sciences/Applied Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory. Attempt* **all** *the parts of Question No.* **1** *and any* **two** *parts from Question No.* **2** *to Question No.* **7.**

1. (a) Test for convergence

$$\int_0^\infty \sin x^2 \, dx \qquad (5)$$

(b) Describe explicitly the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$T(1,2) = (2,3)$$
$$T(0,1) = (1,1) \qquad (5)$$

(c) Find mass centre of a cubical box with no lid, the sides and bottom being made of the same thin material. (6)

P.T.O.

2. (a) Prove that a necessary and sufficient condition for the integrability of a bounded function f defined on $[a, b]$ is that to every $\varepsilon > 0$, there exists a partition ρ of $[a, b]$ for which $u(\rho, f) - L(\rho, f) < \varepsilon$. (8)

(b) Discuss the convergence of the following integrals :–

(i) $\int_1^\infty \frac{x^2}{\sqrt{x^5 + 1}}\, dx$ (ii) $\int_0^1 \frac{dx}{\sqrt{1 - x^3}}$ (8)

(c) Prove that

$$\int_0^\infty \sqrt{y}\, e^{-y^2}\, dy \times \int_0^\infty \frac{e^{-y^2}}{\sqrt{y}}\, dy = \frac{\pi}{2\sqrt{2}} \quad (8)$$

3. (a) Show that

$$\int_C \frac{x^2 dy - y^2 dx}{x^{5/3} + y^{5/3}} = \frac{3\pi}{16}\, a^{4/3}$$

where C is the quarter of the astroid $x = a\cos^3 t$, $y = a\sin^3 t$ from the point $(a, 0)$ to the point $(0, a)$. (8)

(b) Show that

$$\int_0^1 dx \int_0^1 \left(\frac{x^2 - y^2}{x^2 + y^2}\right) dy = \int_0^1 dy \int_0^1 \left(\frac{x^2 - y^2}{x^2 + y^2}\right) dx \quad (8)$$

(c) Evaluate the following line integral by changing it to a double integral using Green's Theorem :

$$\int_C (xy + x + y)dx + (xy + x - y)dy$$

where C is the circle $x^2 + y^2 = a^2$. (8)

4. (a) Give an example of a non commutative infinite ring that does not have a unity. Justify your answer. (8)

(b) Show that intersection of two ideals of a ring is an ideal of the ring. Give an example to show that union of two ideals may not be an ideal. (8)

(c) Show that a commutative ring $(R,+,\cdot)$ is an integral domain iff for all $a, b, c \in R$, $(a \neq 0)$

$$a \cdot b = a \cdot c \Rightarrow b = c \quad (8)$$

5. (a) Define a basis of a vector space and show that (1, 1, 0), (2, 1, 1) and (3, 0, 3) do not form a basis of R^3. (8)

(b) If V and U are vector spaces, $\{v_1, v_2, - - -, v_n\}$ is a basis of V and $w_1, w_2, - - -, w_n$ any arbitrary vectors in U, then there exists a unique linear mapping $T : V \rightarrow U$ such that

$$T(v_i) = w_i, \quad 1 \leq i \leq n \quad (8)$$

(c) Define kernel, range, rank, nullity of a linear transformation. Find kernel, range, rank, nullity of linear transformation $T : R^2 \rightarrow R^3$ defined by

$$T(x, y) = (x + y, x - y, y) \quad (8)$$

6. (a) A uniform bar AB, 10 ft long is hinged at B and supported in a vertical plane by a light string AC which connects A to a point C, 10 ft directly above B. If AB weighs 20 lb and AC = 15 ft, find the tension in AC and the reaction at B. (Assume that the weigt of AB acts through its middle point.) (8)

(b) A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping takes place. (8)

(c) If forces of magnitudes P, Q, R act at a point parallel to and in the direction of sides BC, CA and AB respectively of a ΔABC. Prove that the magnitude of resultant is

$[P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C]$ (8)

7. (a) Derive equation of motion of simple pendulum and show that it is simple harmonic motion. Also find its time period. (8)

(b) Mud is thrown off from the tyre of a wheel (radius a) of a car travelling at a speed V where $V^2 > ga$. Neglecting the resistance of air, show that no mud can rise higher than a height $a + \frac{V^2}{2g} + \frac{ga^2}{2V^2}$ above the ground. (8)

(c) A bead of mass m slides on a smooth wire in form of a parabola with axis vertical and vertex downward. If the bead starts from rest at an end of latus rectum (of length 4p), find the speed with which it passes through the vertex. Find the reaction of wire on bead at this point. (8)