[This question paper contains 4 printed pages.]

Your Roll No. ....

5159 **B**

B.Sc. (Prog.)/III

MP – 301 : MATHEMATICS – II

(Admissions of 2008 and onwards)

Time : 3 *Hours* *Maximum Marks* : 112

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

UNIT – I

1. (a) State Cauchy's convergence criterion for sequences. Hence or otherwise show that the sequence $<a_n>$ given by

$$a_n = 1 - \frac{1}{2!} + \frac{1}{3!} - \ldots\ldots + \frac{(-1)^{n+1}}{n!} + \ldots$$

is a Cauchy's sequence. 7

(b) Prove that $\lim_{n\to\infty} x^n = 0$ if and only if $|x| < 1$. 7

(c) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2 + x_n}$ for all $n \in$ IN, the set of natural numbers. Show that $< x_n >$ converges and find the limit. 7

2. (a) (i) State necessary condition for convergence for a positive term series.

(ii) Test for convergence the following series :

(A) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

(B) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$. 7

(b) Test for convergence the series

$$1-\frac{x^2}{2!}+\frac{x^4}{4!}-......+(-1)^n\frac{x^{2n}}{(2n)!}+.....$$ 7

(c) Show that energy absolutely convergent series is convergent but converse need not be true. 5,2

UNIT - II

3 (a) Solve the differential equation : 11

$(D^3 + D^2 + D + 1)\, y = \sin 2x$.

(b) Solve : 11

$y\,(x^2y^2 + 2)\,dx + x\,(2 - 2x^2y^2)\,dy = 0$

(c) Using the method of variation of parameters, solve : 11

$$\frac{d^2y}{dx^2} + 4y = e^x.$$

4. (a) Solve : 11

$$\frac{a\,dx}{(b-c)yz} = \frac{b\,dy}{(c-a)zx} = \frac{c\,dz}{(a-b)xy}$$

(b) Solve : 11

$px\,(z - 2y^2) = (z - qy)\,(z - y^2 - 2x^3)$

(c) An 8 lb weight is placed upon the lower end of a coil spring suspended from the ceiling. The weight comes to rest is its equilibrium position, thereby stretching the spring 6 inches. The weight is then pulled down 3 inches below its equilibrium position and released at t = 0 with an initial velocity of 1ft./sec., directed downward. Neglecting the resistance of the medium and assuming that no external forces are present, determine the amplitude, period and frequency of the resulting motion. 11

UNIT - III

5. (a) State Cayley Hamilton theorem and verify it for the matrix $\begin{pmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 3 & 2 & -2 \end{pmatrix}$ 10

(b) Let G = $\left\{ \begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix} / a \in R, a \neq 0 \right\}$

Show that G is a group under matrix multiplication. 10

(c) Prove that a non-empty subset H of a group G is a subgroup of G iff $ab^{-1} \in H, \forall a, b \in H$. 10

6. (a) Find eigen values and eigen vectors for the martix.

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$$ 10

(b) Define centre of a group G and prove that it is a normal subgroup of G. 10

(c) State and prove Lagrange's theorem for finite groups. 10