[This question paper contains 4 printed pages.]

1929

Your Roll No. ....

B.Sc. Prog. / III **E**

MATHEMATICS

MA-302 – Analysis, Algebra and Mechanics

(For Physical Sciences/Applied Sciences)

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
Attempt all parts of Question No. **1** *and any* **two** *parts from Question No.* **2** *to Question No.* **7**. **All** *questions carry equal marks.*

1. (a) Define a Riemann integral of a bounded function on [a,b].

(b) Show that the set {5n : n □ Z} is a commutative ring with respect to usual addition and multiplication.

(c) (i) State Laws of static and kinetic friction.

P.T.O.

(ii) A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

2. (a) Show that a bounded function is Riemann integrable on [a,b] if and only if for every □ > 0 there exist partition P of [a,b] such that

$$U(P,f) - L(P,f) < □$$

(b) A function f is defined on [0,1] by

$$f(x) = 1/n \qquad 1/(n+1) < x \leq 1/n \qquad n = 1,2,3.......$$

Prove that f is Riemann integrable on [0.1] and evaluate $\int_0^1 f(x)dx$.

(c) Show that $\int_a^\infty \frac{dx}{x^n}(a>0)$ converges if and only if $n>1$. Hence show that $\int_1^\infty \frac{\tan^{-1}x}{x^2}dx$ is convergent.

3. (a) Show that

$$\sqrt{\pi}\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma(m+1/2)$$

Hence obtain the value of Γ(1/4). Γ(3/4)

(b) Find the value of $\int_c (x^2+y^2)dy$, taken in the counter clockwise sense along the quadrilateral with vertices (0,0), (2,0), (4,4) and (0,4).

(c) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the domain bounded by $y=x^2$ and $y^2=x$.

4. (a) Define a field. Show that ring Z_p of integers modulo p is a field if and only if p is prime.

(b) Define characteristics of a ring. Prove that characteristic of an integral domain is either zero or a prime number.

(c) Define Ideal of a ring R. if A and B are two ideals of a ring R, Show that A+B is an ideal of R generated by $A \cup B$.

5. (a) Prove that

(i) Every subset of a linearly independent set is linearly independent.

(ii) Every superset of a linearly dependent set is linearly dependent.

(b) Show that mapping $T: R^2 \to R^2$ defined by $T(x,y) = (y,x)$ is a linear transformation but mapping

$S: R^2 \to R^2$ defined by $S(x,y) = (1+x,y)$ is not a linear transformation.

(c) Let V be a vector space of dimension n and $T: V \to V$ be a linear transformation such that Range T = Ker T Prove that n is even. Give an example of such linear transformation.

6. (a) Two beads of weight w and w' can slide on a smooth circular wire in a vertical plane. They are connected by a light string which subtends an angle 2β at the centre of the circle when beads are in equilibrium on the upper half of the wire. Prove that inclination α of the string to the horizontal is given by
$\tan \alpha = (w \sim w'/w+w')\tan\beta$

(b) Define mass centre of a system of particles. Establish its existence and uniqueness.

(c) Define Potential energy of the conservative system of particles at a configuration A. Find the Potential energy of a particle attracted towards a fixed point by a force of magnitude $\frac{k^2}{r^n}$, r being the distance from the fixed point, k and n are any constants.

7. (a) Find the expressions for radial and transverse components of velocity and acceleration of a particle moving along a plane curve $r=f(\theta)$.

(b) Show that motion of simple pendulum is simple harmonic motion. Also find its time period.

(c) A particle is projected from a fixed point o with a velocity u making an angle α with horizontal. If R is horizontal range and h is its greatest height, show that

$$u = \left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}$$