[This question paper contains 4 printed pages.]

1964 *Your Roll No.* ....

B.Sc. (Prog.) / III **E**

MP – 301 : MATHEMATICS – II

(Admissions of 2008 and onwards)

Time : 3 Hours *Maximum Marks : 112*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question **1** *is compulsory.*
Attempt any **two** *parts from question* **2** *of* **Unit I.**
Attempt any **two** *questions from* **Unit II**
and any **two** *questions from* **Unit III.**

1. (a) Prove that a Cauchy sequence is bounded. (4)

(b) Let $G = \{1, \omega, \omega^2)$, where ω is the cube root of unity. Prove that G is a Group under multiplication. (4)

(c) Solve the differential equation

$(D^2 + 3D + 2)y = e^x \sin x$ (4)

UNIT I

2\. (a) State and prove Limit comparison test for infinite series. (3,9)

(b) State Monotone convergence Theorem. Prove that the sequence $<x_n>$ given by $x_1 = 1$, $x_{n+1} = \frac{4+3x_n}{3+2x_n}$, for all $n \geq 1$ is convergent. Also find its limit. (2,7,3)

(c) Examine the convergence of the following series

(i) $\sum_{n=1}^{\infty}\left(\sqrt[3]{n^3+1} - n\right)$

(ii) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (6,6)

UNIT II

3\. (a) Solve

$(2y.\sin x.\cos x + y^2\sin x)dx + (\sin^2 x - 2y\cos x)dy = 0.$ (10)

(b) Solve $(x^2D^2 - 4xD + 6)y = x^4.$ (10)

4\. (a) State the Lagrange's method for solving quasilinear partial differential equation of first order. Solve

$p + 2q = 5z + \tan(y - 2x)$ (2,8)

(b) Solve

$$\frac{dx}{dt} = 7x - y,$$

$$\frac{dy}{dt} = 2x + 5y \qquad (10)$$

5. (a) Find the complete integral of the partial differential equation

$$pq = px + qy \qquad (10)$$

(b) A 16lb weight is attached to the lower end of a coil spring suspended from the ceiling, the spring constant of the spring being 10lb/feet. The weight comes to rest in the equilibrium position. Beginning at t=0 an external force given by F(t) =5cos2t is applied to the system. Determine the resulting motion if the damping force in pounds is numerically equal to $2\frac{dx}{dt}$ where $\frac{dx}{dt}$ is the instantaneous velocity in feet per second. (10)

UNIT III

6. (a) Find eigen values and the corresponding eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \qquad (9)$$

(b) State Cayley – Hamilton theorem and verify it for the matrix

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$ (9)

7. (a) Prove that a finite semi-group in which cancellation laws hold is a group. (9)

(b) Prove that every cyclic group is abelian. Is the converse true ? Justify. (9)

8. (a) Let H be a subgroup of G. Prove that H is normal in G if and only if product of two right cosets of H in G is again a right coset of H in G. (9)

(b) If G is a finite group then show that O(a) divides O(G), for all $a \in G$. (9)