[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 288 **C** **Roll No............**

Unique Paper Code : 236362

Name of the Paper : OR-III : Mathematical Programming

Name of the Course : B.Sc. (Mathematical Sciences)

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** questions carry equal marks.

1. (a) Let f(X) be differential in its domain. If f(X) is defined in an open convex set S then show that f(X) is convex if and only if
$f(X_2) - f(X_1) \geq (X_2 - X_1)^T \nabla f(X_1)$ for all $X_1, X_2 \in S$ (8)

(b) Verify whether the following function is convex or concave and find the relative maximum and minimum (if any) solution point of the following function :
$f(X) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$ (7)

2. (a) Use one dimensional search procedure to interactively solve approximately the following equation

$$\text{Max } Z = f(x) = 6x - x^2$$

Use an error tolerance $\varepsilon = 0.04$ and initial bounds as $\underline{x} = 0$ and $\bar{x} = 4.8$. (8)

(b) Use dichotomous search method to find the maximum of each of the following function :–

$$f(x) = \begin{pmatrix} 4x & ; & 0 \leq x \leq 2 \\ 4-x & ; & 2 \leq x \leq 4 \end{pmatrix}$$

Assume $\Delta = 0.05$. (Attempt maximum four steps) (7)

3. (a) Solve the following problem by using Golden Section search method –

$$\text{Min } f(x) = x^4 - 15x^3 + 72x^2 - 1135x \qquad 1 \leq x \leq 15$$

Use $\varepsilon = 0.5$. (Attempt maximum four steps) (8)

P.T.O.

(b) Discuss the Constrained Derivative (Jacobian) method for solving a general non-linear programming problem with equality constraints. (7)

4. (a) Show three iterations of the solution of the following two variable problem by Gradient search procedure –

$$\text{Max } Z = f(x) = 2x_1x_2 + 2x_2 - x_1^2 - 2x_2^2$$

X = (1/2, 1/2) may be taken as the starting trial solution. (9)

(b) Describe 0–1 integer programming problem and formulate the fixed charge problem as a integer programming problem. (6)

5. (a) Solve the following non-linear programming problem by using Lagrangian multiplier technique –

$$\begin{aligned} &\text{Optimize } Z = 4x_1 + 9x_2 - x_1^2 - x_2^2 \\ &\text{Subject to} \quad 4x_1 + 3x_2 = 15 \\ &\qquad\qquad\quad 3x_1 + 5x_2 = 14 \quad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Check the sufficient condition also. (7)

(b) Use Kuhn Tucker conditions to solve the following NLPP –

$$\begin{aligned} &\text{Max} \quad Z = 7x_1^2 - 6x_1 + 5x_2^2 \\ &\text{Subject to} \quad x_1 + x_2 \leq 10 \\ &\qquad\qquad\quad x_1 - 3x_2 \leq 9 \quad x_1, x_2 \geq 0 \end{aligned}$$

(8)

6. By deriving the necessary Kuhn Tucker conditions and using Wolf's method solve the following quadratic programming problem –

$$\begin{aligned} &\text{Max} \quad Z = 2x_1 + x_2 - x_1^2 \\ &\text{subject to} \quad 2x_1 + 3x_2 \leq 6 \\ &\qquad\qquad\quad 2x_1 + x_2 \leq 4 \quad x_1, x_2 \geq 0 \end{aligned}$$

(15)

7. (a) Solve the following non-linear programming problem –

$$\begin{aligned} &\text{Max} \quad Z = 4x_1 - x_1^3 + 2x_2 \\ &\text{Subject to} \quad x_1 + x_2 \leq 1 \quad x_1, x_2 \geq 0 \end{aligned}$$

(6)

(b) Solve the following integer programming problem –

$$\begin{aligned} &\text{Max} \quad Z = x_1 + x_2 \\ &\text{Subject to} \quad 3x_1 + 2x_2 \leq 5 \\ &\qquad\qquad\quad x_2 \leq 2 \\ &\qquad\qquad\quad x_1, x_2 \geq 0 \quad \text{and } x_1 \text{ is an integer} \end{aligned}$$

(9)