

This question paper contains 3 printed pages.

3256

Your Roll No.

B.Tech. / I

J

Paper - EME-101

MATHEMATICS - I

Time : 3 hours

Maximum Marks : 70

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Attempt any five questions.

All questions carry equal marks.

1. (a) Discuss the nature of the following series:

$$(i) \frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots$$

$$(ii) 1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \frac{x^3}{3^2} + \dots$$

(b) If $x_r = \cos \left(\frac{\pi}{2^r} \right) + i \sin \left(\frac{\pi}{2^r} \right)$, show that

$$x_1 \cdot x_2 \cdot x_3 \dots \infty = -1.$$

2. (a) Determine the value of λ for which the system of equations:

P.T.O.

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + 2x_2 + x_3 = -2$$

$$x_1 + x_2 + (\lambda - 5)x_3 = \lambda$$

(i) has no solution

(ii) has a unique solution.

(b) Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

3. (a) Find the asymptotes of the curve $x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 2 = 0$.

(b) Find the Fourier series to represent $x \sin x$ from $x = -\pi$ to $x = \pi$.

4. Solve the following differential equations:

(a) $(D^2 - 1)y = \cosh x \cos x$

(b)
$$\begin{cases} \frac{d^2x}{dt^2} - 3x - 4y = 0 \\ \frac{d^2y}{dt^2} + x + y = 0 \end{cases}$$

5. (a) Evaluate:—

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1-x^2-y^2-z^2}} dz dy dx.$$

(b) Evaluate the following integral by changing the order of integration:

$$\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y} dy dx.$$

6. (a) For the curve $r^2 = a^2 \cos 2\theta$, show that:

$$\rho = \frac{a^2}{3r}.$$

(b) Show that the length of the curve $y = \log \sec x$ between the points $x=0$ and $x=\pi/3$ is $\log(2+\sqrt{3})$.

7. (a) Given $\sin 30^\circ = \frac{1}{2}$, use Taylor's Theorem to evaluate $\sin 31^\circ$ correct to four significant digits given that $\cos 30^\circ = 0.8660$.

(b) If $x+y=1$, prove that:

$$D^n(x^n y^n) = n! [y^n - \binom{n}{1} y^{n-1} x + \binom{n}{2} y^{n-2} x^2 - \dots + (-1)^n x^n].$$