This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7982**

Unique Paper Code : **2511104** **F-1**

Name of the Paper : **Engineering Mathematics [DC–1.2]**

Name of the Course : **B.Tech. Instrumentation**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory. Attempt *Five* questions in all.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Find the unit vector normal to the surface $xy^3z^2 = 4$ at a point $(-1, -1, 2)$. 3

(*b*) If $F(w)$ be the Fourier transform of $f(x)$ and $G(w)$ that of $f(x + a)$, then show that $G(w) = e^{-iaw} F(w)$. 3

(*c*) State Dirichlet conditions for Fourier series. 3

(*d*) Find $L^{-1}\left\{\dfrac{s+2}{s^2 - 4s + 13}\right\}$, where L^{-1} represents the inverse Laplace operator. 3

P.T.O.

(e) Find the Z-transform of unit impulse $\delta(k)$ and $\frac{a^k}{k!}$. 3

2. (a) Prove :

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\nabla^2 \vec{A} + \vec{\nabla}(\vec{\nabla} . \vec{A}).$$ 4

(b) Show that :

$$\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + (3xz^2)\hat{k}$$

is a conservative vector field. Hence find the scalar potential. 7

(c) Calculate $\nabla^2\phi$ when $\phi = 3x^2z - y^2z^3 + 4x^3y + 2x - 3y - 5$ at the point (1, 1, 0). 4

3. (a) A sinusoidal voltage E sin (wt), is passed through a half wave rectifier which clips the negative half cycle of the wave. Find the Fourier series of the resulting periodic function :

$$f(t) = \begin{cases} 0 & ; -L < t < 0 \\ E \sin(wt); & 0 < t < L \end{cases}$$

Here the period $T = 2L = 2\pi/w$. 7

(*b*) The following values x give the displacement in cms of a certain machine part of the rotation x of the flywheel. Expand $f(x)$ in the form of a Fourier series upto second harmonic : 5

x^o	$f(x)$
0	0
30	9.2
60	14.4
90	17.8
120	17.3
150	11.7

(*c*) Find the function $f(x)$ if its Fourier sine transform is given by $\frac{e^{-as}}{s}$. 3

4. (*a*) Using Laplace transform find the solution of the initial value problem 5

$$y''(x) - 4y'(x) + 4y(x) = 64\sin 2x;$$

$$y(0) = 0 \text{ and } y'(0) = 1.$$

(*b*) Using Laplace transform evaluate the integral : 5

$$\left(\int_0^{\infty} te^{-3t} \sin t \, dt\right).$$

(*c*) State and prove the convolution theorem for Laplace's transform. 5

5. (*a*) Find the Z-transform of $f(t)$, where $f(t) = \begin{cases} 5^k & k < 0 \\ 3^k & k \geq 0 \end{cases}$. 5

(*b*) Find :

$$Z^{-1}\left(\frac{1}{(z-5)^3}\right), \text{ when } |z| > 5$$

and determine the region of convergence. 7

(*c*) Show $Z[a^k f(k)] = F(z/a)$. 3

6. (*a*) Differentiate between ordinary and partial differential equations. Give *one* example of each related to a physical problem. 2

(*b*) Prove that :

$$\sinh^{-1}(x) = \log\left(x + \sqrt{x^2 + 1}\right).$$ 5

(*c*) Prove that the half range sine series of the function $x(\pi - x)$ in the range $0 < x < \pi$ is given by

$$x(\pi - x) = \frac{8}{\pi}\left[\frac{\sin x}{1^2} + \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} + \ldots\right]$$

Hence deduce

$$\sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi}{945}.$$ 8

7. (*a*) Find the Fourier cosine transform of $f(x) = e^{-ax}$ for $x \geq 0$ and $a > 0$. Hence deduce the integral

$$\left(\int_0^{\infty} \frac{\cos sx}{s^2 + a^2} ds\right).$$ 5

(*b*) Obtain Inverse Laplace transform of the following function using convolution theorem

$$L^{-1}\left(\frac{1}{(s^2 + 1)^3}\right).$$ 5

(*c*) Evaluate $Z(2n + (\cos\theta + i\sin\theta)^n + (n + 1)^2)$. 5