(This paper contains 3 printed pages)

Roll No.:

Sr. No. of Question Paper: 1773 F-3

Unique Paper Code: 2511104

Name of Course: B.Tech. Instrumentation

Name of Paper: Engineering Mathematics

Semester: I

Time : 3 Hours *Maximum Marks : 75*

Question No. 1 is compulsory. Attempt any Five Questions.

Use of non-programmable scientific calculator is allowed.

Q.1 (a) Find the nature of the vector $\vec{V} = (x+3y)\hat{\imath} + (y-3z)\hat{\jmath} + (x-2z)\hat{k}$, whether it is conservative, solenoidal or both. (3)

(b) If $F(\alpha)$ be the Fourier transform of $f(t)$, find the Fourier transform of $\{f(t+a)\}$, where 'a' is a constant (3)

(c) Under what conditions can a function $f(x)$be expanded in a Fourier series? To what value does the series converge at a point of discontinuity? (3)

(d) Find $L\left(e^{2t}\cos 3t + \frac{4}{3}e^{2t}\sin 3t\right)$where L represents the Laplace operator (3)

(e) Find the Z-transform of discrete unit step function $u(k)$ (3)

Q.2 (a) Prove $\vec{\nabla}.(\phi\vec{\nabla} \times \vec{A}) = (\vec{\nabla} \times \vec{A}).\vec{\nabla}\phi$ (4)

(b) A fluid motion is given by $\vec{A} = (x^2 - yz)\hat{\imath} + (y^2 - xz)\hat{\jmath} + (z^2 - xy)\hat{k}$. Is the motion irrotational? If so, find its velocity potential. (7)

(c) Verify that V satisfies the Laplace equation $\nabla^2 V = 0$, where $V = e^{3x+4y}cos5z$ (4)

Q.3 (a) Find the Fourier series of the function $f(x) = \begin{cases} 0 & -\pi < x \le 0 \\ x & 0 < x \le \pi \end{cases}$. (5)

(b) The displacement of a part of a machine is tabulated with corresponding angular moment 'x' of the crank. Express $f(x)$ as a Fourier Series upto second harmonic

x^{o}	0	30	60	90	120	150
$f(x^{\circ})$	1.80	1.10	0.30	0.16	0.50	1.30
x^{o}	180	210	240	270	300	330
$f(x^{\circ})$	2.16	1.25	1.30	1.52	1.76	2.00

(5)

(c) Find the Fourier Sine transform of $f(x) = e^{-ax}$ for $x \ge 0$ and $a > 0$. Hence deduce the integral (5)

$$\left(\int_0^{\infty} \frac{s.\, sinsx}{s^2 + a^2} ds\right)$$

Q.4 (a) Using Laplace transform find the solution of the initial value problem (5)

$$y''(t) + 25y(t) = 10\, cos5t\,;\ y(0) = 2\ and\ y'(0) = 0$$

(b) Show that the Laplace transform of a piecewise continuous function $f(x)$ with period T is given by $L[f(x)] = \frac{1}{1-e^{-sT}} \int_0^T e^{-sx} f(x)\, dx$ (5)

(c) Obtain Inverse Laplace transform of the following function using convolution theorem $L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right)$ (5)

Q.5 (a) Find the Z-transform of $f(t)$ where $f(t) = \begin{cases} 10^k & k < 0 \\ 7^k & k \ge 0 \end{cases}$ (7)

and determine the region of convergence

(b) Find $Z^{-1}\left(\frac{1}{(z-3)(z-2)}\right)$ when $|z| < 2$ (5)

(c) State any six properties of Z-Transforms (3)

Q.6 (a) The temperature at any point in space is given by $T = xy + yz + zx$. Determine the derivative of T in the direction of the vector $3\hat{\imath} - 4\hat{k}$ at the point (1,1,1) (3)

(b) Prove that $\cosh^{-1}(x) = \log\left(x + \sqrt{x^2 - 1}\right)$ (5)

(c) Prove that the half range cosine series of the function $x(\pi - x)$ in the range $0 < x < \pi$ is given by (7)

$$x(\pi - x) = \frac{\pi^2}{6} - \left[\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \cdots\ldots\right]$$

Hence deduce

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

Q.7 (a) If $f(x)$ is an even function, derive the fourier coefficients, and hence write its Fourier expansion (5)

(b) A periodic square wave function (t) , in terms of unit step function is (5)

$$f(t) = k[u_0(t) - 2u_a(t) + 2u_{2a}(t) - 2u_{3a}(t) + \ldots\ldots]$$

Evaluate Laplace transform of $f(t)$

(c) Using convolution theorem, find the inverse Z-transform of $\left(\frac{z}{(z-a)}\right)^3$. (5)

Deduce for $\left(\frac{z}{(z-1)}\right)^3$