[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 7801 **F-2** **Your Roll No...............**

Unique Paper Code : 2341201

Name of the Course : **B.Tech. in Computer Science**

Name of the Paper : Linear Algebra for Comp. Sc.

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. **1** is compulsory.
4. All the symbols have their usual meanings.

1. (a) Find the inverse of the following matrix using elementary row transformations

$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$$

(b) Describe the null space of the matrix $A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 8 & 6 & 16 \end{bmatrix}$.

(c) State and prove Cauchy-Schwarz's inequality.

(d) Use Cramer's rule to solve Ax = b, where

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(e) Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(f) Let M be the space of all n × n matrices. Let T: M → M be a map such that $T(A) = \frac{A - A^t}{2}$. Show that T is linear. Also show that the kernel of T consists of the space of symmetric matrices.

(g) The fraction of rental cars in a city starts at 0.02. The fraction outside that city is 0.98. Every month, 80% of the cars stay in the city (and 20% leave). Also 5% of the outside cars come in the city (95% stay outside). Estimate the fraction of cars coming and leaving the city at the end of 5 months. (5×7=35)

2. (a) Find the complete solution to Ax = b, where

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 \\ 0 & 0 & 1 & 4 \\ 1 & 3 & 1 & 6 \end{bmatrix} \text{ and } b = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}$$

(b) Write the LU-decomposition of the matrix

$$A = \begin{bmatrix} 3 & -6 & -3 \\ 2 & 0 & 6 \\ -4 & 7 & 5 \end{bmatrix}$$

Also compute E_{21}^{-1}, E_{31}^{-1}, and E_{32}^{-1} to find L. (5,5)

3. (a) Define column space and null space of a matrix A. Let Ax = b be the system of m linear equations in n unknowns. Prove that the column space and null space of A are the subspaces of R^m and R^n respectively.

(b) Find the bases and dimensions for all four fundamental subspaces of the following matrix A, namely, the column space of A, the null space of A, the row space of A and the null space of A^T. Also determine the rank of the matrix A.

$$A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix}$$ (5,5)

4. (a) Use Gram-Schmidt's orthogonalisation process to find the QR decomposition of the matrix

$$W = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 2 \end{bmatrix}$$

(b) Find the pivots of the given matrix A and verify that det.A = product of the pivots.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$ (5,5)

5. (a) Diagonalise the following matrix A and hence find A^3.

$$A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$$

(b) Find the singular value decomposition (svd) of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$.

(5,5)

6. (a) Define a convex set. Draw the region in the xy-plane where $x + 2y \leq 6$, $2x + y \leq 6$ and $x \geq 0$ $y \geq 0$. Which corner minimizes the cost $c = 2x - y$?

(b) Test whether the following matrix is positive definite or not.

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(5,5)

7. (a) Find the best line $b = C + Dt$ to fit $b = 0,8,8,20$ at times $t = 0,1,3,4$.

(b) Let V be the vector space generated by the three functions $f_1(x) = 1$, $f_2(x) = x$, $f_3(x) = x^2$, where $x \in R$. Let $D : V \to V$ be the derivative. What is the matrix of D w.r.t. the basis $\{f_1, f_2, f_3\}$? (5,5)