Sl. No. of Q.P. 2287

Unique Paper Code: 2511204

Name of the Paper: Engineering Mathematics - I

Name of the Course: B.Tech. Electronics

F-4

Semester: II

Duration: 3 hours

Maximum Marks: 75

Instructions for candidates

Q. No. 1 is Compulsory.
Attempt Five Questions in all.

Question 1: **Compulsory Question** **[5 × 3 = 15]**

a) Show that $\bar{\nabla} \cdot (\bar{\nabla} \times \bar{A}) = 0$.

b) For what value of λ the system of homogeneous equations have non-trivial solutions.

$$\begin{aligned} 2x + y + 2z &= 0 \\ 4x + 3y + \lambda z &= 0 \\ x + y + 3z &= 0 \end{aligned}$$

c) Evaluate the integral $\oint \frac{e^z}{z+1}\, dz$, $C : \left|z + \frac{1}{2}\right| = 1$.

d) Determine for what values of x ,the series are convergent

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x)^{2n-1}}{(2n-1)!}.$$

e) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction 2i-j-2k

Question 2

a) State Gauss Divergence theorem. Evaluate [5]

$$\oiint \boldsymbol{F} . \hat{n}\ ds$$

where $\mathbf{F} = 4xz\,\mathbf{i} - y^2\,\mathbf{j} + yz\,\mathbf{k}$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0$ and $z = 1$.

b) Prove [5]

i) $\vec{\nabla}\, |\vec{r}|^3 = 3\, r\, \vec{r}$

ii) $\vec{\nabla} . (r^3 \vec{r}) = 6\, r^3$

iii) Find the expression for vector differential operator in orthogonal curvilinear coordinates. [5]

Question 3

a) Solve the system of equations using Gauss Elimination method. [5]

$$\begin{aligned} 8y + 2z &= -7 \\ 3x + 5y + 2z &= 8 \\ x + 2y + 8z &= 26. \end{aligned}$$

b) Find the eigenvalue and eigenvectors of the matrix, $A = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$ [5]

c) Prove the Cayley – Hamilton theorem for the above matrix. [5]

Question 4

a) State Cauchy's general principle of convergence and find the convergence of the sequence: [5]

$$\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-\dots\dots$$

b) Use D' Alembert's ratio test [5]

$$\left(\left[x^{2n}\cdot\frac{n}{(n^2+1)}\right]^{1/2}\right)$$

c) Use Cauchy's nth root test for x>0 [5]

$$\sum\frac{x^{2n}}{2^n}.$$

Question 5

a) Using the Cauchy – Riemann equation, show that f(z) = $|z|^2$ is not analytical at any point [5]

b) Integrate the function f(z) around C [5]

$$\int_C \frac{e^z}{z^2(z+1)^3}\,dz\ ,\ C: |z| = 2$$

c) Find all values of z which satisfies e^z = 1 + i [5]

Question 6

a) Verify the Green's theorem in plane to evaluate $\oint_c A.dr$, given vector A=(xy+y²)dx+x²dy and C is the closed curve of the region bounded by y = x and y=x². [5]

b) Prove that any Hermitan matrix is a sum of Hermitian and Skew - Hermitian matrix. [5]

c) Find an equation for the tangent plane to the surface 2xz²-3xy-4x=7 at the point (1,-1,2). [5]

Question 7

a) Find the interval of convergence for [5]

i) Exponential series

ii) Binomial series

b) Show that the function f(z) = $\frac{z^4+2z+1}{z^2+5z+2}$ has a pole of the order 2 at z = ∞. [5]

c) Compute the residues at singular point [5]

$$f(z)=\frac{z}{(z+1)(z-2)}$$