This question paper contains 3 printed pages.

3262 *Your Roll No............................*

B.Tech (M) / II

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EME - 201
Paper - MATHEMATICS - II

Time : 3 hours *Maximum Marks : 70*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions. Assume missing data, if any. All questions carry equal marks.

1 a) Find the constants a, b, c so that

$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$

is irrotational.

If $\vec{F} = \text{grad } \phi$, show that

$\phi = \frac{x^2}{2} - \frac{3y^2}{2} + z^2 + 2xy + 4xz - yz$

where ϕ is known as scalar potential.

b) Evaluate $\iint_s \vec{A}.\hat{n}\, ds$, where $\vec{A} = z\hat{i} + x\hat{j} + 3y^2 z\hat{k}$

and s is the surface of the cylinder $x^2 + y^2 = 1$ included in the first octant between $z = 0$ and $z = 2$.

2 a) Prove that

$n P'_n(x) - P'_{n-1}(x) = n P_n(x)$

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b) Show that

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi} \text{Cos}\,(n\theta - x\,\text{Sin}\theta)\,d\theta.$$

3 a) Find the Laplace Transform of the Periodic function

$$f(t) = \frac{kt}{T}\text{; for } 0 < t < T, f(t+T) = ft$$

b) Solve the following differential equation by using Laplace Transform :

$$\frac{d^2y}{dx^2} + y = 3\,\text{Cos}2x$$

where $y = \frac{dy}{dx} = 0$ at $x = 0$.

4 a) Evaluate

(i) $L^{-1}\left(\frac{s+2}{(s^2 - 4s + 13)}\right)$

(ii) $L^{-1}\left[\frac{1}{s\,(s^2 + a^2)}\right]$

b) Define unit step function and find the laplace transform at the following :

$$f(t) = \begin{cases} t - 1, & \text{if } 1 < t < 2 \\ 0, & \text{otherwise} \end{cases}$$

5 a) Find the bilinear transform which maps the point $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $|z| < 1$.

b) Verify Cauchy's theorem for the integral of z^3 taken over the boundary of rectangle with vertices 1, 1, $1 + i$, $-1 + i$,

6 a) Evaluate the following integral by Contour integration:

$$\int_0^{2\pi} \frac{d\theta}{(5 - 3 \sin\theta)^2}$$

b) Show that

$$\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta = \frac{1}{2}\,\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = \frac{\pi}{\sqrt{2}}$$

7 a) Use Stoke's theorem to evaluate

$$\iint_S (\nabla \times \vec{F}) . \hat{n}\, ds.$$

where $\vec{F} = y\,\hat{i} + (x - 2xz)\,\hat{j} - xy\,\hat{k}$ and s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the x y plane.

b) Show that $\iiint_V (\nabla . \vec{F})\, dV = \frac{8}{3}$

where $\vec{F} = (2x^2 - 3z)\,\hat{i} - 2xy\,\hat{j} - 4x\,\hat{k}$ and V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.