This question paper contains 3 printed pages.

8477 Your Roll No. ....

B. Tech. (EE/EC) / II A

PAPER : EEE/EEC-201— MATHEMATICS – II

Time : 3 hours *Maximum Marks* : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions. All questions carry equal marks. Assume missing data, if any.

1. (*a*) Show that $\vec{F}=(2xy+z^3)\mathbf{i}+x^2\mathbf{j}+3z^2x\mathbf{k}$ is a conservative field. Find its scalar potential and also the work done in moving a particle from (1, –2, 1) to (3, 1, 4).

(*b*) Show that

$$\iiint \nabla\times\vec{F}\,dV=\frac{8}{3}(\mathbf{j}-\mathbf{k})$$

where $\vec{F}=(2x^2-3z)\mathbf{i}-2xy\mathbf{j}-4x\mathbf{k}$ and V is the closed region bounded by the planes $x=0$, $y=0$, $z=0$ and $2x+2y+z=4$.

2. (*a*) Verify Stokes' theorem for the function :

$$\vec{F}=x^2\mathbf{i}+xy\mathbf{j}$$

integrated round the square whose sides are $x=0$, $y=0$, $x=a$ and $y=a$ in the plane $z=0$.

(*b*) A vector field is given by:—

$$\vec{F}=(\sin y)\mathbf{i}+x(1+\cos y)\mathbf{j}.$$

Evaluate the line integral over the circular path $x^2+y^2=a^2, z=0$.

3. (*a*) Show that:—

$$\left[J_0(x)\right]^2+2\left[J_1(x)\right]^2+2\left[J_2(x)\right]^2+\ \ldots\ =1$$

(*b*) Determine the values of $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. 7

4. (*a*) If $L[f_1(t)]=F_1(s)$ and $L[f_2(t)]=F_2(s)$, then show that:

$$L\int_0^t f_1(x)f_2(t-x)\,dx=F_1(s)\cdot F_2(s).$$

(*b*) Find the solution of initial value problem, by using Laplace transform:—

$$y''+9y=6\cos 3t$$

given $y(0)=2$ and $y'(0)=0$.

5. (*a*) Find the Laplace transform of the periodic function:

$$f(t)=t \qquad 0<t<c$$
$$=2c-t, \quad c<t<2c$$

(*b*) Evaluate:

(*i*) $L^{-1}\left[\dfrac{s-1}{s^2-6s+25}\right]$

(ii) $$L\left[\int_0^{\infty} te^{-3t}\sin t\,dt\right].$$

6. (a) Find the image of the infinite strip:—

(i) $\frac{1}{4}<y<\frac{1}{2}$

(ii) $0<y<\frac{1}{2}$

under the mapping function $w=\frac{1}{z}$. Depict the regions so obtained graphically and interpret.

(b) Evaluate $\int_0^{1+i}(x^2+iy)\,dz$, along the path $y=x$.

7. (a) Evaluate the following by Cauchy's integral formula:

(i) $\int_C \frac{e^z\,dz}{(z+1)^2}$, where C is $|z-1|=3$

(ii) $\int_C \frac{\sin^2 z}{(z-\pi/6)^3}\,dz$, where C is $|z|=1$.

(b) Show that:—

$$\int_0^{\infty} x^{y-1}e^{-x}(\log x)^n\,dx=\frac{d^n}{dy^n}(\Gamma y)$$