This question paper contains 3 printed pages.

8452 Your Roll No. ....

B. Tech. (M) / II **A**

PAPER : EME-201— MATHEMATICS – II

Time : 3 hours *Maximum Marks* : 70

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions. All questions carry equal marks. Assume missing data, if any.

1. (*a*) Find the values of the constants a, b and c so that the maximum value of the directional derivative of

$$\varphi = axy^2 + byz + cz^2x^3$$

at (1, 2, −1) has a magnitude 64 and is in the direction parallel to the axis of z.

(*b*) Show that:—

$$\nabla\left\{\vec{a}\cdot\nabla\left(\frac{1}{r}\right)\right\} = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a}\cdot\vec{r})\vec{r}}{r^5}$$

2. (*a*) Use Divergence theorem to evaluate:—

$$\iint_S \vec{F}\cdot\hat{n}\,dS,$$

where $\vec{F} = 4x^3\mathbf{i} - x^2y\mathbf{j} + x^2z\mathbf{k}$ and S is the surface bounding the region $x^2+y^2=a^2$, $z=0$, $z=b$.

(*b*) Apply Stokes' theorem to evaluate:—

$$\int_C \left[(x+y)\,dx+(2x-z)\,dy+(y+z)\,dz\right]$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6) and equation of plane $3x+2y+z=6$ passes through these points.

3. (*a*) Show that:—

$$\int_0^1 x\,J_n(\alpha x)\,J_n(\beta x)\,dx=0,$$

where α and β are roots of $J_n(x)=0$.

(*b*) Prove that:—

$$\int_{-1}^{1}\left[P_n(x)\right]^2 dx=\frac{2}{2n+1}.$$

4. (*a*) Find the solution of initial value problem, by using Laplace transform

$$y''+y=\sin 3t.$$

given that $y(0)=0$ and $y'(0)=0$.

(*b*) If $f(t)$ is a periodic function, then prove that:—

$$L\left[f(t)\right]=\frac{1}{1-e^{-sT}}\int_0^T e^{-st} f(t)\,dt$$

where T is a period.

5. (*a*) Find:—

$$L\left[\int_0^t e^{-2t}\, t \sin^3 t \, dt\right].$$

(*b*) Evaluate:—

(*i*) $L^{-1}\left(\dfrac{s}{s^2+4s+13}\right)$

(*ii*) $L\left(\dfrac{1-\cos at}{t}\right)$.

6. (*a*) Determine the analytic function $w=u+iv$, if

$$v=\log(x^2+y^2)+x-2y.$$

(*b*) Evaluate the integral:—

$$\int_0^{1+i} (x-y+ix^2)\, dz$$

along the path $y=x$.

7. (*a*) Evaluate the integral:—

$$\int_0^{2\pi} \frac{d\theta}{(a+b\sin\theta)},\ a>|b|$$

by contour integration.

(*b*) Show that:—

$$\iint_D x^{m-1} y^{n-1}\, dx\, dy = \frac{\Gamma m\, \Gamma n\, a^{m+n}}{\Gamma(m+n+1)}$$

where D is the domain $x \geq 0$, $y \geq 0$ and $x+y \leq a$.