[This question paper contains 4 printed pages.]

**Sr. No. of Question Paper : 1678** **F-3** **Your Roll No...............**

Unique Paper Code : 2352601

Name of the Course : **B.Tech. (Comp. Sc.) Allied Courses**

Name of the Paper : Numerical Methods

Semester : III

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Use of Scientific Calculator is allowed.

1. (a) Define the Round-off error. If we have three-digit arithmetic, compute

   (0.99 + 0.0044) + 0.0042 and 0.99 + (0.0044 + 0.0042). Compare the results using significant digits. (6)

   (b) Perform five iterations by Bisection method to find the square root of 7 in (2, 3). (6)

   (c) Apply Secant method to $x^3 + x^2 - 3x - 3 = 0$ to determine an approximation to a root lying in the interval (1, 2). Perform four iterations. (6)

2. (a) Use Newton's method to solve the given non-linear system of equations :

$$f(x,y) = x^2 + y^2 - 1 = 0$$

$$g(x,y) = x^2 - y = 0.$$

   Take initial approximation $(x_0, y_0) = (0.5, 0.5)$ and perform three iterations. (6½)

(b) Solve the following system of equations by using Gauss elimination (row pivoting) method.

$$3x - y + 2z = 7$$
$$x + y + 2z = 9$$
$$2x - 2y - z = -5$$

(6½)

(c) Find the inverse of the following matrix using Gauss–Jordan elimination method

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 3 & 8 & 14 \end{bmatrix}$$

(6½)

3. (a) Use Gauss-Seidel method to solve given system of equations :

$$2x_1 - x_2 + x_3 = -1$$
$$x_1 + 2x_2 - x_3 = 6$$
$$x_1 - x_2 + 2x_3 = -3$$

Take initial approximation as $X^{(0)} = (0,0,0)^T$ and perform three iterations. (6)

(b) For the function $f(x) = \ln(x)$, construct Lagrange form of the interpolating polynomial for f(x) passing through the points (1, *ln*1), (2, *ln*2) and (3, *ln*3). Use the polynomial to estimate *ln*(1.5) and *ln*(2.4). What is the error in each approximation ? (6)

(c) Define the forward difference operator $\Delta$, backward difference operator $\nabla$ and central difference operator $\delta$. Prove that :

(i) $\delta = \nabla(1-\nabla)^{-1/2}$

(ii) $\Delta\left(\frac{1}{f_i}\right) = \frac{-\Delta f_i}{f_i f_{i+1}}$ (6)

4. (a) For the following data, calculate the differences and obtain the backward difference polynomial.

| x | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |
|---|---|---|---|---|---|
| f(x) | 1.40 | 1.56 | 1.76 | 2.0 | 2.28 |

Interpolate at x = 0.25. (6½)

(b) Obtain the piecewise linear interpolating polynomials for the function f(x) defined by the data

| x | 1 | 2 | 4 | 8 |
|---|---|---|---|---|
| f(x) | 3 | 7 | 21 | 73 |

Hence estimate the values of f(3) and f(7). (6½)

(c) Derive three-point forward difference formula :

$$f'(x_i) \approx \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{x_{i+2} - x_i} \text{ with error } O(h^2)$$

Approximate f′(1), f′(2) and f′(3) from the following data :

| x | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| f(x) | 2 | 4 | 8 | 16 | 32 |

(6½)

5. (a) Derive Trapezoid rule with its error term. What is the degree of precision for this method ? (6)

(b) Compute $\int_0^2 e^{-x^2} dx$ using Gaussian quadrature. (6)

(c) Apply Euler's method to approximate the solution of the initial value problem :

$$\frac{dy}{dx} = -yx^2, \qquad y(1) = 2$$

over the interval [1, 2] using 5 steps. (6)

6. (a) Given the initial value problem :

$$\frac{dy}{dx} = -2xy^2, \qquad y(0) = 1$$

Estimate y(0.4), using Ralston method, with h = 0.2. (6½)

(b) Solve the initial value problem :

$$\frac{dy}{dx} = 1 - 2xy, \qquad y(0) = 0$$

with h = 0.5 on the interval [0, 1] using classical $4^{th}$ order Runge-Kutta method. (6½)

(c) Apply finite-difference method to solve the problem :

$$\frac{d^2y}{dx^2} = y + x, \qquad 0 \le x \le 1$$

with y(0) = 2, y(1) = 2.5, and h = 0.25. (6½)