This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 1677

Unique Paper Code : 2362301 F-3

Name of the Paper : **Introduction to Operational Research & Linear Programming**

Name of the Course : **B.Tech. (Computer Science) Allied Courses**

Semester : **III**

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions carry equal marks.

There are three sections in the paper.

All sections are compulsory.

Attempt any *five* questions from each Section.

Use of simple calculator is allowed.

Section A

1. Define the term O.R. and write its applications in different areas of real life.

2. Explain any *two* of the following :

   (*a*) Slack variables

   (*b*) Artificial variables

   (*c*) Dual prices.

P.T.O.

3. What do you mean by linearly dependent and linearly independent vectors ? Check whether the given set $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}1\\1\\-1\end{bmatrix}$ is linearly dependent or linearly independent.

4. What do you mean by extreme point in convex set ? Find all extreme points from the system of equations given below :

$$2x_1 + x_2 - x_3 = 2$$

$$3x_1 + 2x_2 + x_3 = 3$$

5. (*a*) Define convex set. Does the union of convex sets is a convex set ?

(*b*) Show that the following set is convex :

$$S = \{(x_1, x_2) \mid x_1 + x_2 \leq 3, x_1 \geq 0, x_2 \geq 0\}.$$

6. Consider the following LPP with two variables :

Maximize $Z = 2x_1 + 3x_2$

Subject to :

$$2x_1 + x_2 \leq 4$$

$$x_1 + 2x_2 \leq 5$$

$$x_1, x_2 \geq 0.$$

(*a*) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

(*b*) Verify graphically that the solution obtained in (*a*) is the optimum LP solution. Hence, conclude that the optimum solution can be determined algebraically by considering the basic feasible solutions only.

Section B

7. Day Trader wants to invest a sum of money that would generate an annual yield of at least $ 10000. Two stocks groups are avilable : blue chips and high tech, with average annual yields of 10% and 25%, respectively. Though high-tech stocks provide higher yield, they are more risky, and Trader wants to limit the amount invested in these stocks to no more than 60% of the total investment. Formulate the above problem as a LPP to maximize an annual yield.

8. Solve the given LPP graphically :

Maximize $Z = x_1 + 2x_2$

Subject to :

$$x_1 - x_2 \leq 1$$

$$x_1 + 2x_2 \geq 3$$

$$x_1, x_2 \geq 0.$$

9. Consider the following LP :

Maximize $Z = x_1$

Subject to :

$$5x_1 + x_2 = 4$$
$$6x_1 + x_3 = 8$$
$$3x_1 + x_4 = 3$$
$$x_1, x_2, x_3, x_4 \geq 0.$$

(*a*) Solve the problem by inspection (do not use the Gauss-Jordan row operations), and justify the answer in terms of the basic solutions of the Simplex method.

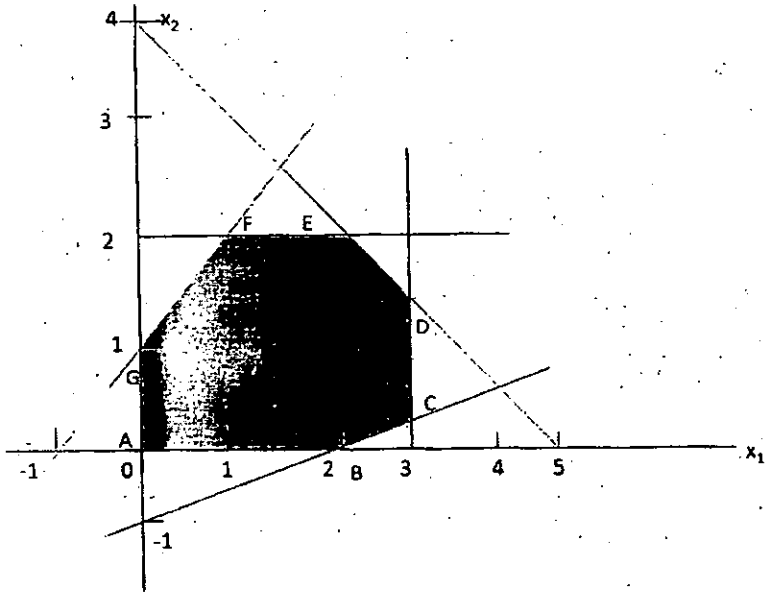
(*b*) Repeat (*a*) assuming that the objective function calls for minimizing $z = x_1$.

10. Consider the two-dimensional solution space in figure given below.

Suppose that the objective function is given as

Maximize $z = 6x_1 + 3x_2$.

If the simplex iterations start at point A, identify the path to the optimum point D.



Determine the entering variable, the corresponding ratios of the feasibility condition, and the change in the value of z.

11. For the following LP, identify three alternative optimal basic solutions and then write a general expression for all the non-basic alternative optima :

Maximize $Z = 2x_1 + 4x_2$

Subject to :

$$x_1 + 2x_2 \leq 5$$

$$x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

12. Use Big-M method to solve :

Maximize $Z = 12x_1 + 20x_2$

Subject to :

$$6x_1 + 8x_2 \geq 100$$

$$7x_1 + 12x_2 \geq 120$$

$$x_1, x_2 \geq 0.$$

Section C

13. Comment on the future of artificial variables at the optimal table of phase 1.

14. Write the dual of the following LPP :

Minimize $Z = x_1 + x_2 + x_3$

Subject to :

$$x_1 - 3x_2 + 4x_3 = 5$$

$$x_1 - 2x_2 \leq 3$$

$$2x_2 - x_3 \geq 4.$$

$x_1, x_2 \geq 0$ and x_3 is unrestricted.

15. What do you understand by feasibility and optimality ranges of the variables in LPP ?

16. Consider the following LP model :

Maximize $Z = 4x_1 + 10x_2$

Subject to :

$$2x_1 + x_2 \leq 50$$

$$2x_1 + 5x_2 \leq 100$$

$$2x_1 + 3x_2 \leq 90$$

$$x_1, x_2 \geq 0$$

Check the optimality and feasibility of each of the following basic solution :

Basic variables = (x_1, x_2, x_5), Inverse = $\begin{vmatrix} 5/8 & -1/8 & 0 \\ -1/4 & 1/4 & 0 \\ -1/2 & -1/2 & 1 \end{vmatrix}$

17. Use dual simplex method to solve the given LPP :

Minimize $Z = 3x_1 + x_2$

Subject to :

$$x_1 + x_2 \geq 1$$

$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0.$$

18. Consider the following LP :

Maximize $Z = 5x_1 + 12x_2 + 4x_3$

Subject to :

$$x_1 + 2x_2 + x_3 \leq 10$$

$$2x_1 - x_2 + 3x_3 = 8$$

$$x_1, x_2, x_3 \geq 0.$$

Using x_4 and A as starting variables, the optimal tableau is given as :

Basic	x_1	x_2	x_3	x_4	A	Solution
Z	0	0	3/5	29/5	–2/5+M	274/5
x_2	0	1	–1/5	2/5	–1/5	12/5
x_1	1	0	7/5	1/5	2/5	26/5

Write the associated dual problem and determine its optimal solution.