[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1597 **F-3** **Your Roll No................**

Unique Paper Code : 2351303

Name of the Course : **B.Sc. (H) Mathematics**

Name of the Paper : Numerical Methods

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Marks are indicated against each question.
5. Use of Scientific Calculator is allowed.

1. (a) Discuss the rate of convergence of the Newton Raphson method and give the geometrical interpretation of the method.

   (b) Find an interval in which the smallest positive root of $f(x) = x^4 - 3x^2 + x - 10 = 0$ lies. Perform 3 iterations of the Bisection method to find an approximation of a root of the above equation.

   (c) A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0,1)$. Perform 3 iterations of the Secant method to find an approximation of a root of this equation. (13)

2. (a) Show that the initial approximation x_0 for finding $\frac{1}{N}$ for a positive integer N by the Newton-Raphson method must satisfy $0 < x_0 < \frac{2}{N}$ for convergence.

(b) Solve the system of equations :

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

using Gauss Seidel method. Perform 3 iterations taking $x^{(0)} = (0,0,0)$ as the initial approximation.

(c) Solve the system of linear equations :

$$x_1 + x_2 + x_3 = 6$$
$$3x_1 + 3x_2 + 4x_3 = 20$$
$$2x_1 + x_2 + 3x_3 = 13$$

using Gauss Elimination method with partial pivoting. (13)

3. (a) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

using Gauss Jordan method.

(b) Consider the system of linear equations :

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

where 'a' is a real constant. For what value of a, the Gauss Jacobi and Gauss Seidel methods converge ?

(c) Solve the system of linear equations :

$$4x + y + 2z = 4$$
$$3x + 5y + z = 7$$
$$x + y + 3z = 3$$

using Gauss Jacobi method. Perform 3 iterations taking $x^{(0)} = (0,0,0)$ as the initial approximation. (12)

4. (a) Find the interpolating polynomial that fits the data given in the table below :

x	−2	0	1	2
f(x)	−15	7	6	5

Also, find an approximation to f(x) at x = 0.5.

(b) Calculate the nth divided difference of $f(x) = \frac{1}{x}$ based on the points $x_0, x_1, x_2, \ldots, x_n$.

(c) Define operators δ and μ. Prove that

$$\Delta = \frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$$

where all the symbols have their usual meaning. (12)

5. (a) Derive the Trapezoidal rule for approximating the value of a definite integral.

(b) Find an approximate value of the integral $\int_0^1 \frac{dx}{1+x}$ using Simpson's Rule.

(c) Find an approximate value of the integral $\int_1^2 \frac{dx}{x^2}$ using Boole's Rule.

(12)

6. (a) Apply Euler's method to find an approximate solution of the Initial value problem

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \qquad 1 \le t \le 6, \qquad x(1) = 1$$

using 5 steps.

(b) Using Runge-Kutta method of order 4, find an approximate solution for the initial value problem

$$\frac{dx}{dt} = -2tx^2, \qquad 0 \le t \le 0.6, \qquad x(0) = 1$$

Perform 2 steps.

(c) For the set of data tabulated below, find the forward difference interpolating polynomial :

x	−1	0	1	2
f(x)	5	1	1	11

Also find the approximate value of f(x) at x = −0.5. (13)