[This question paper contains 2 printed pages.]

Sr. No. of Question Paper : 1699   F-3   Your Roll No................

Unique Paper Code   : 2352501

Name of the Course   : **B.Tech. Polymer Sciences : Allied Course**

Name of the Paper   : Differential Equations and Mathematical Modelling

Semester   : III

Duration : 3 Hours   Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. Question No. 1 is compulsory.
4. Use of non-programmable calculator is permitted.

1. (a) Find the wronskian of the functions $x^2 - x$, $3x^2 + x + 1$, $9x^2 - x + 2$. State whether these are linearly dependent.

   (b) Solve $(xy^2 + x)dx + (yx^2 + y)dy = 0$.

   (c) Classify the ordinary points and regular singular points of the differential equation $(1 - x^2)y'' + 2y' - 3y = 0$.

   (d) Find the orthogonal trajectories of $xy = c$.

   (e) Show that the family of spheres $(x - a)^2 + (y - b)^2 + z^2 = r^2$

   satisfies the first order partial differential equation $z^2(p^2 + q^2 + 1) = r^2$.

   $(3 \times 5 = 15)$

2. (a) A 120 gallon tank initially contains 90 lb of salt dissolved in 90 gal of water Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full ?

   (b) If a substance cools from 370K to 330K in 10 minutes, when the temperature of the surrounding air is 290K, find the temperature of the substance after 40 minutes.

P.T.O.

(c) Find an integrating factor and solve
$$2xydx + 3x^2dy = 0.$$
(5×3=15)

3. (a) Solve $2xyy' = y^2 - 2x^3$, $y(1) = 2$.

(b) In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, how many may be expected at the end of 12 hours.

(c) Solve the differential equation $y'' + y' - 6y = 39 \cos 3x$, by the method of undetermined coefficients. (5×3=15)

4. (a) Solve the differential equation $y'' + 6y' + 9y = 0$.

(b) Solve the differential equation $y'' + 5y' + 4y = 0$, by factorizing the differential operator.

(c) Solve the differential equation $x^2y'' + 9xy' + 25y = 0$. (5×3=15)

5. (a) Solve the differential equation $y'' + 4y = \sec 2x$ by the method of variation of parameters.

(b) Solve the differential equation $y''' - 7y' - 6y = 0$.

(c) Solve the simultaneous differential equations,
$$x' = 7x - y$$
$$y' = 2x + 5y$$
(5×3=15)

6. (a) Solve in series the differential equation $y'' + x^2y = 0$.

(b) Solve in series the differential equation $9x(1 - x)y'' - 12y' + 4y = 0$.

(c) Express the polynomial $f(x) = 1 - 3x + 3x^2$ in terms of Legendre polynomials.
(5×3=15)

7. (a) Use the separation of variables to solve the equation $u_x + 2u_y = 0$.

(b) Reduce the following equation into canonical form and find the general solution
$$yu_x + u_y = x.$$

(c) Find the characteristic equations and then reduce the following equation to the canonical form.
$$y^2u_{xx} - x^2u_{yy} = 0.$$
(5×3=15)