Sl. No. Of Question Paper: 6286

Unique Paper Code	:	2352601
Name of the Paper	:	Numerical Methods
Name of the Course	:	B.Tech. (Comp. Sc.) (FYUP) Allied Courses
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75 Marks

F-5

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions are compulsory.
4. Use of Scientific Calculator is allowed.

1.(a) Define the floating point number system. Add, subtract and multiply the given floating point numbers 0.123×10^3 and 0.456×10^2. Write the results in three digits mantissa form. (6)

(b) Perform five iterations by Bisection method to find the square root of 3 in $[1, 2]$. (6)

(c) Apply Newton-Raphson's method to find the positive real zero of the following function

$$f(x) = x^3 - 9x + 2$$

Take initial approximation $x_0 = 0.5$ and perform four iterations. (6)

2.(a) Use Newton's method to solve the given non-linear system of equations:

$$f(x, y) = x^2 + 4y^2 - 16 = 0$$

$$g(x, y) = xy^2 - 4 = 0.$$

Take initial approximation $(x_0, y_0) = (1, 2)$ and perform three iterations. $(6\frac{1}{2})$

(b) Solve the following system of equations by using Gauss elimination (row pivoting) method: $(6\frac{1}{2})$

$$2x + 6y + 10z = 0$$

$$x + 3y + 3z = 2$$

$$3x + 14y + 28z = -8$$

(c) Find the inverse of the following matrix using Gauss –Jordan elimination method: $(6\frac{1}{2})$

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

3. (a) Use Jacobi method to solve the given system of equations:

$$2x_1 - x_2 + x_3 = -1$$

$$x_1 + 2x_2 - x_3 = 6$$

$$x_1 - x_2 + 2x_3 = -3$$

Take initial approximation as $X^{(0)} = (0, 0, 0)^T$ and perform three iterations. (6)

(b) Construct the divided difference table for the following data set and write the Newton form of the interpolating polynomial: (6)

x	-7	-5	-4	-1
$f(x)$	10	5	2	10

(c) Define the forward difference operator Δ, central difference operator δ and averaging operator μ.

Prove that :

(i) $\mu = \sqrt{(1 + \frac{1}{4}\delta^2)}$

(ii) $\Delta(f_i\, g_i) = f_i\, \Delta g_i + g_{i+1}\, \Delta f_i$ (6)

4. (a) For the following data, calculate the differences and obtain the forward difference polynomial.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.0	2.28

Interpolate at $x = 0.35$. $(6\frac{1}{2})$

(b) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data

x	-1	0	1	2
$f(x)$	$\frac{1}{3}$	1	3	9

Hence estimate the values of $f(-0.7)$ and $f(1.4)$. $(6\frac{1}{2})$

(c) Derive central difference formula :

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{x_{i+1} - x_{i-1}} \text{ with error } O(h^2)$$

Use this formula to apply Richardson extrapolation to approximate $f'(3)$ with error $O(h^4)$ from the following data $(6\frac{1}{2})$

x	1	2	3	4	5
$f(x)$	2	4	8	16	32.

5. (a) State Simpson's rule and apply it to approximate: (6)

$$\frac{\pi}{4} = \int_0^1 \frac{1}{1+x^2} dx$$

(b) Compute $\int_1^2 \frac{1}{x} dx$ using Romberg integration (up to second level of extrapolation). (6)

(c) Apply Euler's method to approximate the solution of the initial value problem:

$$\frac{dy}{dx} = x + y, \qquad y(0) = 2$$

over the interval $[0, 2]$ using 5 steps. (6)

6.(a) Given the initial value problem:

$$\frac{dy}{dx} = -2xy^2, \qquad y(0) = 1$$

estimate $y(0.4)$, using Heun method, with $h = 0.2$. $(6\frac{1}{2})$

(b) Solve the initial value problem:

$$\frac{dy}{dx} = 3x^2y, \qquad y(0) = 1$$

with $h = 0.5$ on the interval $[0, 1]$ using classical 4^{th} order Runge-Kutta method. $(6\frac{1}{2})$

(c) Apply finite-difference method to solve the problem:

$$\frac{d^2y}{dx^2} = y + x(x - 4), \qquad 0 \le x \le 4$$

with $y(0) = y(4) = 0$, and step size $h = 1$. $(6\frac{1}{2})$