Sl. No. of Question Paper: 6285

Unique Paper Code	:	2362301
Name of the Paper	:	Introduction to Operational Research & Linear Programming
Name of the Course	:	B.Tech (Computer Science)
Semester	:	III
Duration	:	3 hours
Maximum Marks	:	75 Marks

F-5

Instructions for Candidates:

1) All questions carry equal marks.
2) There are three sections in the paper. All sections are compulsory.
3) Attempt any five questions from each section.
4) Use of simple calculator is allowed.
5) Total numbers of printed pages are five.

SECTION A

Q1. Discuss the significance of O.R. in decision making and the role of computers in this field?

Q2. Explain any two of the following:

a) Shadow prices

b) Artificial variables

c) Surplus variables

Q3. What do you mean by linearly dependent and linearly independent vectors? Check whether the given set $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\1\end{bmatrix}, \begin{bmatrix}8\\7\end{bmatrix}$ is linearly dependent or linearly independent.

Q.4 What do you mean by extreme point in convex set? Find all extreme points from the system of equations given below:

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

Q5. a) Define convex set? Does the intersection of convex sets is a convex set?

b)Show that the following set is convex:

$$S = \{(x_1, x_2) |\, x_1{}^2 + x_2{}^2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$$

Q. 6 Consider the following LPP with two variables

$Maximize\ z = 2x_1 + 3x_2$

Subject to

$x_1 + 3x_2 \leq 12$

$3x_1 + 2x_2 \leq 12$

$x_1,\ x_2 \geq 0$

a) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

b) Verify graphically that the solution obtained in (a) is the optimum LP solution. Hence, conclude that the optimum solution can be determined algebraically by considering the basic feasible solutions only.

SECTION B

Q7. John must work at least 20 hours a week to supplement his income while attending school. He has the opportunity to work in two retail stores. In store 1, he can work between 4.5 and 12 hours a week, and in store 2, he is allowed between 5.5 and 10 hours. Both stores pay the same hourly wage. In deciding how many hours to work in each store, John wants to base his decision on work stress. Based on interviews with present employees, John estimates that, on ascending scales of 1 to 10, the stress factors are 8 and 6 at stores 1 and 2, respectively. Because stress mounts by the hour, he assumes that the total stress for each store at the end of the week is proportional to the number of hours he works in the store. Formulate the given problem as LPP to get the number of hours required for john work in each store.

Q8. Solve the given LPP graphically

$Maximize\ z = 3x_1 + 2x_2$
Subject to

$2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12$
$x_1,\ x_2 \geq 0$

Q9. Consider the following system of equations:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + 5x_4 + x_5 &= 8 \\ 5x_1 - 2x_2 + 6x_4 + x_6 &= 16 \\ 2x_1 + 3x_2 - 2x_3 + 3x_4 + x_7 &= 6 \\ -x_1 + x_3 - 2x_4 + x_8 &= 0 \\ x_1, x_2, \ldots x_8 &\geq 0 \end{aligned}$$

Let $x_5, x_6, \ldots,$ and x_8 be given initial basic feasible solution. Suppose that x_1 becomes basic. Which of the given basic variables must become non basic at zero level to guarantee that all the variables remain nonnegative, and what is the value of x_1 in the new solution? Repeat this procedure for x_2, x_3 and x_4.

Q10. The following tableau represents a specific simplex iteration. All variables are nonnegative. The tableau is not optimal for either maximization or minimization. Thus, when a non basic variable enters the solution, it can increase or decrease z or leave it unchanged, depending on the parameters of the entering non basic variables.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	solution
z	0	-5	0	4	-1	-10	0	0	620
x_8	0	3	0	-2	-3	-1	5	1	12
x_3	0	1	1	3	1	0	3	0	6
x_1	1	-1	0	0	6	-4	0	0	0

a) Categorize the variables as basic and non-basic, and provides the current values of all the variables.

b) Assuming that the problem is of the maximization type, identify the non basic variables that have the potential to imporve the value of z . If each such variable enters the basic solution. Determine the associated leaving variable. If any, and the associated change in z. Do not use the Gauss-Jordan row operations.

Q11. For the following LP, identify three alternative optimal basic solutions and then write a general expression for all the non basic alternative optima.

$$Maximize\ z = 6x_1 + 4x_2$$

Subject to

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

Q12. Use Big-M method to solve

$$Maximize\ z = 2x_1 + 3x_2$$

Subject to

$x_1 + 2x_2 \leq 4$

$x_1 + x_2 = 3$

$x_1, x_2 \geq 0$

SECTION C

Q13. Comment on the future of artificial variables at the optimal table of Phase 1.

Q.14 Write the dual of the following primal problem

$Minimize\ z = x_1 - 3x_2 - 2x_3$

Subject to

$3x_1 - x_2 + 2x_3 \leq 7$

$2x_1 - 4x_2 \geq 12$

$-4x_1 + 3x_2 + 8x_3 = 10,$

$x_1, x_2 \geq 0\ and\ x_3$ is unrestricted.

Q.15 Explain the effect of changing the resource vector and cost vector individually on optimal solution.

Q16. Consider the following LP model:

$Maximize\ z = 4x_1 + 10x_2$

Subject to

$2x_1 + x_2 \leq 50$

$2x_1 + 5x_2 \leq 100$

$2x_2 + 3x_2 \leq 90$

$x_1, x_2 \geq 0$

Check the optimality and feasibility of each of the following basic solution.

Basic Variables $=(x_1, x_2, x_5)$, Inverse$=\begin{bmatrix} 5/8 & -1/8 & 0 \\ -1/4 & 1/4 & 0 \\ -1/2 & -1/2 & 1 \end{bmatrix}$

Q.17 Use dual simplex method to solve the given LPP

$Minimize\ z = -2x_1 - x_2$

Subject to

$3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0.$

Q.18 Consider the following LP:

$Maximize\ z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to

$x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

Using x_3 and x_4 as starting variables, the optimal tableau is given as

Basic	x_1	x_2	x_3	x_4	Solution
Z	2	0	0	3	16
x_3	0.75	0	1	-0.25	2
x_2	0.25	1	0	0.25	2

Write the associated dual problem and determine its optimal solution.