[This question paper contains 4 printed pages]

Sr. No. Of Question Paper: 6300 **Roll No.....**

Unique Paper Code: 2352501(Paper Code:- 438)

Name of the Course: **B.Tech. Polymer Sciences (FYUP) Allied Course**

F-5

Name of the Paper: **Differential Equations and Mathematical Modelling**

Semester: III

Duration:3 Hours

Maximum Marks:75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt five questions in all.
3. Question No. **1** is compulsory.
4. Use of non-programmable calculator is permitted.

1 (a) Solve $(x+1)\frac{dy}{dx} = x(y^2+1)$.

(b) Find the wronskian of the functions x^2-2x, $3x^2+x+2$, $4x^2-x+1$. State whether these are linearly dependent.

(c) Classify the ordinary points and regular singular points of the differential equation $x^2(x^2-4)y'' + 2x^3y' - 3y = 0$.

(d) Find the orthogonal trajectories of the curve $y^2 = cx$.

(e) Show that a family of spheres $x^2 + y^2 + (z-c)^2 = r^2$

satisfies the first-order linear partial differential equation $yp - xq = 0$. (3X5=15)

2. (a) A tank contains 1000 gal of water in which 200lb of salt is dissolved. 50 gallons of brine, each gallon containing (1+cost)lb of dissolved salt, runs into the tank per minute. The mixture, kept uniform by stirring, runs out at the same rate. Find the amount of salt y(t) in the tank at any time t.

(b) A body of temperature 80° F is placed in a room of constant temperature 50° F at time t = 0. At the end of 5 minutes, the body has cooled to a temperature of 70° F. Find the temperature of body at the end of 10 minutes.

(c) Reduce to exact form and solve

$$2\sin(y^2)\,dx + xy\cos(y^2)\,dy = 0;\ \ y(2) = \sqrt{\frac{\pi}{2}} \qquad \text{(5X3=15)}$$

3. (a) Solve $3y' + xy = xy^{-2}$.

(b) A bacterial population is known to have a rate of growth proportional to the number present. If between noon and 2 p.m. the population triples, at what time, no controls being exerted, should B become 100 times what it was at noon.

(c) Solve the differential equation $2y'' - y' - 3y = x^3 + x + 1$, by the method of undetermined coefficients. (5X3=15)

4. (a) Solve the differential equation $y'' + 2y' + 2y = 0$

(b) Solve the differential equation $y'' - 4y' - 5y = 0$, by factorizing the differential operator.

(c) Solve the differential equation $x^2y'' - xy' - 3y = 0$. (5X3=15)

5. (a) Solve the differential equation $y'' + 4y = tan2x$, by the method of variation of parameters.

(b) Solve the differential equation $y''' - 2y'' - y' + 2y = 0$

(c) Solve the simultaneous differential equations,

$$\begin{aligned} x' + 4x + 3y &= t \\ y' + 2x + 5y &= e^t. \end{aligned} \qquad \text{(5X3=15)}$$

6. (a) Solve in series the differential equation $y'' + xy = 0$.

(b) Solve in series the differential equation

$$2x^2y'' + (2x^2 - x)\,y' + y = 0.$$

(c) Express the polynomial $f(x) = 4x^3 - 2x^2 - 3x + 8$ in terms of Legendre polynomials. (5X3=15)

7 (a) Use the separation of variables to solve the equation $y^2{u_x}^2 + x^2{u_y}^2 = (xyu)^2$.

(b) Reduce the following equation into canonical form and find the general solution

$u_x - u_y = u$.

(c) Find the characteristic equations and then reduce the equation

$$x^2u_{xx} + 2\,xyu_{xy} + y^2u_{yy} = 0$$

to the canonical form. (5X3=15)