This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1440**

Unique Paper Code : **2341303** **F-7**

Name of the Paper : **Discrete Structures**

Name of the Course : **B.Tech. Computer Science (erstwhile FYUP)**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt any *four* questions out of Questions Nos. **2** to **7**.

Parts of a question must be answered together.

1. (*a*) Construct the truth table of the compound proposition : 3

$$(P \vee \neg Q) \rightarrow (P \wedge Q).$$

(*b*) Define bijection mapping. Assume f be the function from

$$\{a, b, c, d\} \text{ to } \{1, 2, 3, 4\}$$

with

$$f(a) = 4, f(b) = 2, f(c) = 1, \text{ and } f(d) = 3.$$

Prove that f is a bijection. 3

(*c*) Find Conjuctive Normal Form for the proposition 4

$$(P \vee Q) \wedge \neg P \rightarrow \neg Q.$$

(*d*) A palindrome is a word that reads the same forward or backward. How many seven letter palindromes can be formed out of English alphabets ? 3

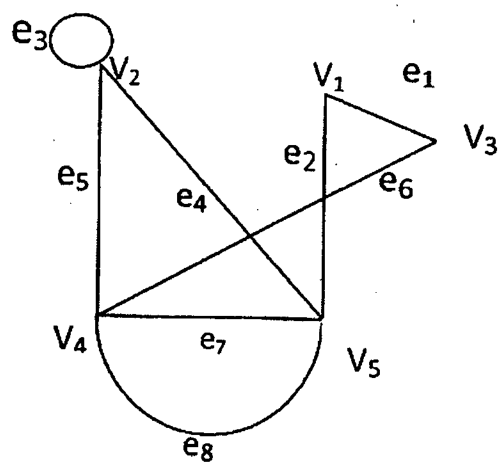
P.T.O.

(*e*) Prove that if $f(x)$ is $O(g(x))$, then 5

$$g(x) = \Omega f(x).$$

(*f*) For a given graph G, prove that the sum of degree of the vertices is always even. 3

(*g*) Find the adjacency matrix of the given multigraph : 4



(*h*) Assume a be a numeric function such that : 5

$$a = \begin{cases} 2 & 0 \le r \le 3 \\ 2^{-r} + 5 & r \ge 4 \end{cases}$$

Determine $S^2 a$ and $S^{-2} a$.

(*i*) Let R be a relation on the set

$$A = \{4, 5, 6, 7\}$$

defined by

$$R = \{(4, 5), (5, 5), (5, 6), (6, 7), (7, 4), (7, 7)\}$$

Find the reflexive, transitive and symmetric closure of R. 3

(*j*) Let g be the function from the set $\{a, b, c\}$ to itself such that

$$g(a) = b, \; g(b) = c, \text{ and } g(c) = a.$$

Let f be the function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ such that

$$f(a) = 3, \; f(b) = 2 \text{ and } f(c) = 1.$$

Find the compositions $f \cdot g$. 2

2. (*a*) Show that :

$$(P \wedge Q) \rightarrow (P \vee Q)$$

is a tautology (without using truth tables.) 5

(*b*) Show that the hypotheses "It is not sunny this afternoon and it is colder than yesterday". "We will go swimming only if it is sunny." "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset." 5

3. (*a*) Let f_1 and f_2 be functions from R to R such that :

$$f_1(x) = x^2 \text{ and}$$

$$f_2(x) = x - x^2.$$

What are the functions $f_1 + f_2$ and $f_1.f_2$? 5

(*b*) Prove by the principle of mathematical induction that : 5

$$P(n) : 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2 = n(n + 1)(2n + 1)/6.$$

4. (*a*) Solve the following recurrence using Master's method : 5

$$T(n) = 125\ T(n/5) + n^3.$$

(*b*) A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month. Find a recurrence relation for the number of pairs of rabbits on the island after *n* months, assuming that no rabbits ever die. 5

5. (*a*) Find the particular solution for the given recurrence relation : 7

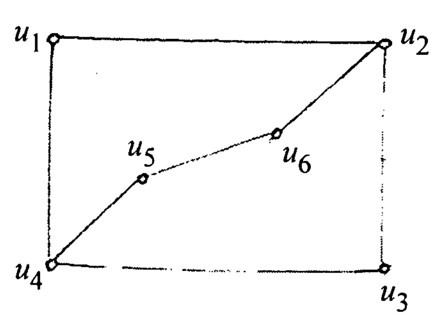
$$a_r = 6a_{r-1} - 11\ a_{r-2} + 6a_{r-3}$$

with the initial conditions

$$a_0 = 2,\ a_1 = 5 \text{ and } a_2 = 15.$$

(*b*) How many edges does a full binary tree with 1000 internal vertices have ? 3

6. (*a*) What are isomorphic graphs ? Show that the given graphs are isomorphic ? 5



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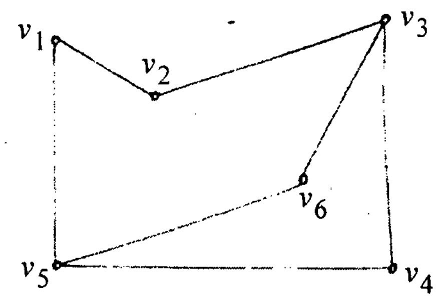

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(*b*) Use Insertion sort to sort the given list of elements 6, 2, 3, 1, 5, 4. 5

7. (*a*) Let P and Q be the propositions : 3

P : It is below freezing.

Q : It is snowing.

Write the following propositions using P, Q and logical connectives :

(*i*) It is below freezing and not snowing.

(*ii*) It is not below freezing and it is not snowing.

(*iii*) If it is below freezing, it is also snowing.

(*b*) Show that n^2 is not $O(n)$. 4

(*c*) Show that the Cartesian product B × A is not equal to Cartesian product A × B, where 3

$$A = \{1, 2\} \text{ and } B = \{a, b, c\}.$$