[This question paper contains 6 printed pages.]

Sr. No. of Question Paper : 1509 **F-7** **Your Roll No...............**

Unique Paper Code : 2362301

Name of the Paper : Introduction of Operational Research and Linear Programming

Name of the Course : **B.Tech Computer Science (Erstwhile FYUP) Allied Course**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Answer **fifteen** questions in all.
3. **All** questions carry equal marks.
4. Simple calculators are allowed.

1. Explain the importance of operational research in decision making. (5)

2. Find all possible basic solutions to the following set of linear equations.

$$2x_1 + 3x_2 + x_3 = 6$$

$$x_1 + 3x_2 + 5x_3 = 5$$

Also check whether any of the above solutions is degenerate or not. (5)

3. Define the basis and dimension of a vector space. Test whether the set of vectors $a_1 = [1, 1, 0]$, $a_2 = [3, 0, 1]$, $a_3 = [5, 2, 1]$ form a basis of R^3 ? (5)

4. Define a convex set. Examine the convexity of the following set :

$S = \{(x_1, x_2): x_1 + x_2 \leq 1, \forall x_1, x_2 \in R\}$ (5)

5. A firm produces three products A, B and C. It uses two type of raw material I and II of which 5,000 and 7,500 units available. The raw material requirements per unit of the products are given below :

Raw Material	Requirement per unit of product		
	A	B	C
I	3	4	5
II	5	3	5

The labour time for each unit of product A is twice that of product B and three times that of product C. The entire labour force of the firm can produce the equivalent of 3,000 units. The minimum demand of the three products is 600, 650 and 500 units respectively. Formulate the problem as a linear programming problem (LPP) that will maximize the profit. (5)

6. Consider the following LPP :

Maximize $Z = 5x_1 + 3x_2$

Subject to : $3x_1 + 5x_2 <= 15$

$5x_1 + 2x_2 <= 10$

$x_1, x_2 >= 0$

(a) Determine all basic solutions of the problem and classify them as feasible and infeasible. (3)

(b) Show how the infeasible basic solutions are represented on the graphical solution space. (2)

7. Use graphical method to solve the following LPP :

Minimise Z = $3x_1 + 4x_2$

Subject to $3x_1 + 4x_2 >= 240$

$2x_1 + x_2 >= 100$

$5x_1 + 3x_2 >= 120$

$x_1, x_2 >= 0$ (5)

8. Use Big M method to solve the following LPP

Maximize Z = $10x_1 + 20x_2$

Subject to $2x_1 + 4x_2 >= 16$

$x_1 + 5x_2 >= 15$

$x_1, x_2 >= 0$ (5)

9. How do you identify following in the optimal simplex table ?

(a) Alternate solution (2)

(b) Unbounded solution (2)

(c) Infeasible solution (1)

10. Solve the following LPP by dual simplex method

Minimize Z = $3x_1 - 2x_2 + x_3$

Subject to $3x_1 + x_2 + x_3 >= 3$

$-3x_1 + 3x_2 + x_3 >= 6$

$x_1 + x_2 + x_3 <= 6$

$x_1, x_2, x_3, x_4 >= 0$ (5)

11. Consider the following LPP

Maximize $Z = 3x_1 + 4x_2 + x_3 + 7x_4$

Subject to $8x_1 + 3x_2 + 4x_3 + x_4 <= 7$

$2x_1 + 6x_2 + x_3 + 5x_4 <= 3$

$x_1 + 4x_2 + 5x_3 + 2x_4 <= 8$

$x_1, x_2, x_3, x_4 >= 0$

Its associated optimal simplex table is given as :

Basic	x_1	x_2	x_3	x_4	x_5	x_6	x_7	Solution
Z	0	169/38	½	0	1/38	53/38	0	83/19
x_1	1	9/38	½	0	5/38	5/38	0	16/19
x_4	0	21/19	0	1	–1/19	–1/19	0	5/19
x_7	0	159/38	9/2	0	–1/38	–1/38	1	126/19

Obtain the variations in cost coefficients which are permitted without changing the optimal solutions. (5)

12. Obtain the dual for the following primal problem :

Minimize $Z = 5x_1 + 6x_2 + x_3$

Subject to $x_1 + 2x_2 + x_3 = 15$

$-x_1 + 5x_2 <= 18$

$4x_1 + 7x_2 <= 20$

$x_1, x_2 >= 0$, x_3 unrestricted (5)

13. Consider the following LPP :

Maximize $Z = 4x_1 + 14x_2$

Subject to $2x_1 + 7x_2 + x_3 = 21$

$7x_1 + 2x_2 + x_4 = 21$

$x_1, x_2, x_3, x_4 >= 0$

Check the optimality and feasibility of the following basic solution :

Basic variables = (x_2, x_4), Inverse of the basis matrix = $\begin{pmatrix} 1/7 & 0 \\ -2/7 & 1 \end{pmatrix}$ (5)

14. Find any three alternate optimal solution (if they exist) for the following LPP :

Maximize $Z = 2x_1 + 4x_2$

Subject to $x_1 + 2x_2 <= 5$

$x_1 + x_2 <= 4$

$x_1, x_2 >= 0$ (5)

15. Consider the LPP :

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to $x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

$x_1, x_2, x_3, x_4 >= 0$

By using x_3 and x_4 as the starting variables, the optimal table is given by

Basic	x_1	x_2	x_3	x_4	Solution
Z	2	0	0	3	16
x_3	3/4	0	1	−1/4	2
x_2	1/4	1	0	1/4	2

Write the associated dual problem, and determine its optimal solution in two ways. (5)

16. Solve the following LPP by Two Phase method :

Maximize $Z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 <= 2$

$3x_1 + 4x_2 =>= 12$

$x_1, x_2 >= 0$ (5)