This question paper contains 3 printed pages. *Your Roll No. ....*

Sl. No. of Ques. Paper	**: 6118**	**F**
Unique Paper Code	**: 2341303**	
Name of Paper	**: Discrete Structures**	
Name of Course	**: B.Tech. Computer Science**	
Semester	**: IV**	
Duration :	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Section A is compulsory. Do any four questions from Section B.

Q.1)

Section-A

a) Give an explicit formula for a function from the set of integers to the set of positive integers that is
   i) one-to-one, but not onto.
   ii) onto, but not one-to-one.
   iii) one-to-one and onto. [3]

b) Show that $3x^2 + 8x$ is $\Theta(x^2)$ [3]

c) Give the symbolic form of the following:
   - The sun is bright and the humidity is not high.
   - The crop will be destroyed if there is a flood.
   - Mark is neither rich nor happy. [3]

d) Use Master theorem to solve the following recurrence:
$$T(n) = 9\,T(n/3) + n$$ [3]

e) Consider the set {a, b, c, d, e, f, g, h, i, j}. In how many ways can we select four letters out of these letters (repetition is not allowed)? [3]

f) Define Hamiltonian path and Hamiltonian circuit. Draw a graph that has an Euler path and Hamiltonian circuit? [3]

g) Find the smallest relation containing the relation {(1,2), (1,4),(3,3),(4,1)} that is reflexive and transitive. [3]

h) Show that every 2-colorable graph is bipartite. [2]

i) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4. How many vertices does it have of degree 1? [3]

j) Find the solution of the recurrence relation
$$a_n - 4a_{n-1} + 4a_{n-2} = 0 \text{ where } a_0 = 0;\ a_1 = 1$$ [3]

k) In a graph the number of odd degree vertices is always even. Prove. [3]

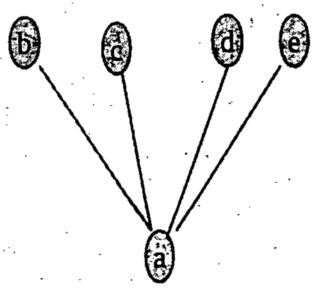
l) Show that at least three of any 25 days chosen must fall in the same month of the year. [3]

Turn over

(Section –B)

1. (a) Prove using mathematical induction that $n^2 - 7n + 12$ is nonnegative whenever n is an integer with $n \geq 3$. [4]

(b) Find matrix of partial order whose Hasse diagram is



[3]

(c) Given A={a, b, c, d}. Consider the relation R on A: R = {(a, a), (b, b), (b, c), (c, b), (d, b), (d, d)}. Is R reflexive? Is it transitive also? Give reasons. [3]

2. (a) Show that a simple graph that has a circuit with an odd number of vertices in it cannot be colored using two colors. [3]

(b) Show that for any connect simple graph ***G*** with 11 vertices either ***G*** is planar or complement of ***G*** is planar. [4]

(c) Prove using mathematical induction that there are at most m^h leaves in an m-ary tree of height h. [3]

3. (a) Show that the solution of $T(n) = 2T(\sqrt{n}) + \lg n$ is $O(\lg n \lg\lg n)$ [4]

(b) How many vertices are there in a graph with 20 edges if each vertex is of degree 5. [3]

(c) Solve the following recurrence relation using Master Theorem:

$$T(n) = 16T(n/4) + n^2$$ [3]

4. (a) Consider the following argument and determine whether it is valid or not. Either I will get good marks or I will not graduate. If I did not gradute I will go to USA. I get good marks. Thus, I would not go to USA. [4]

(b) Write the contrapositive, converse, and inverse of the following statement:

You sleep late if it is Saturday [3]

(c) Determine whether or not the following two propositions are logically equivalent:

$p \rightarrow (\neg q \wedge r)$; $\neg p \vee \neg(r \rightarrow q)$. [3]

5. (a) Find the generating function for the numeric function given by:

1, 1, 2, 2, 3, 3, 4, 4, 5, 5, ... [3]

(b) Let **c=ab.** Show that $\Delta c_r = a_{r+1}(\Delta b_r) + b_r(\Delta a_r)$. [3]

(c) Find the total solution of the recurrence relation

$a_n + 4a_{n-1} = 7$ where $a_0 = 3$ [4]

6. (a) Explain the Θ – notation? Show $6n^3 \neq \Theta(n^2)$. [1+2]

(b) Show that

$$\sum_{k=1}^{n} 1/k$$

bounded above by a constant. [3]

(c) Show that

$$\sum_{k=1}^{\infty} (k-1)/2^k = 0$$

[4]