This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper	**: 6191**	**F-5**
Unique Paper Code	**: 2341501**	
Name of Paper	**: Probability Theory and Statistical Computing**	
Name of Course	**: B.Tech. Computer Science**	
Semester	**: V**	
Duration	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

(इस प्रश्न-पत्र के मिलते ही ऊपर दिये गये निर्धारित स्थान पर अपना अनुक्रमांक लिखिये।)

NOTE:— *Answers may be written either in English or in Hindi; but the same medium should be used throughout the paper.*

टिप्पणी:— इस प्रश्नपत्र का उत्तर अंग्रेज़ी या हिन्दी किसी एक भाषा में दीजिए; लेकिन सभी उत्तरों का माध्यम एक ही होना चाहिए।

Question No. 1 is compulsory and is of 35 marks (7×5 = 35 marks).
Attempt any four questions from Q. No. 2 to 7. (2×5 = 10 marks each)
Parts of a question must be answered together.
Use of Non-Programmable Scientific Calculator is allowed.
The symbols have their usual meaning.

Q.1.

(a) State Baye's theorem. Stores *A*, *B*, and *C* have 50, 75, and 100 employees and respectively 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns and this is a woman. What is the probability that she works in store *C*?

(b) Suppose that an airplane engine will fail, when in flight, with probability 1 − p independently from engine to engine. Suppose that the airplane will make a successful flight if at least 50 percent of its engines remain operative. For what values of p is a four-engine plane preferable to a two-engine plane?

(c) Let the probability density of X be given by

$$f(x) = \begin{cases} c\,(4x - x^2), & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

(i) What is the value of c ?

(ii) Find CDF of X.

(d) Suppose the joint density of X and Y is given by

$$f(x,y) = \begin{cases} 4y(x-y)e^{-(x+y)}; & 0 < x < \infty, 0 \le y \le x \\ 0 \quad ; & otherwise \end{cases}$$

Compute E[X|Y =y].

(e) If X and Y are independent Poisson random variables with respective means λ_1 and λ_2. Calculate the conditional expected value of X given that $X + Y = n$.

(f) Three white and three black balls are distributed in two urns in such a way that each contains three balls. We say that the system is in state i, i = 0, 1, 2, 3, if the first urn contains i white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second and conversely with the ball from the second urn. Let X_n denote the state of the system after the n^{th} step. Calculate the transition probability matrix.

(g) Data was collected on x = shear force (kgs) and y = percent fiber dry weight to test the toughness and fibrousness of asparagus as a major determinant of quality. The following observations were recorded .

$$n = 18, \sum x_i = 1950, \sum x_i^2 = 251970, \sum y_i = 47.92, \sum y_i^2 = 130.6074, \sum x_i y_i = 5530.92$$

Calculate the value of sample correlation coefficient and also compute the coefficient of determination. Interpret the results.

Q.2(a) Bill and George go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with probability 0.7, whereas George, independently, hits the target with probability 0.4.

(i) Given that exactly one shot hit the target, what is the probability that it was George's shot?

(ii) Given that the target is hit, what is the probability that George hit it?

(b) The dice game craps is played as follows. The player throws two dice, and if the sum is seven or eleven, then she wins. If the sum is two, three, or twelve, then she loses. If the sum is anything else, then she continues throwing until she either throws that number again (in which case she wins) or she throws a seven (in which case she loses). Calculate the probability that the player wins.

Q.3(a) Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability $1-p$. The winner of the series is the first team to win four games. Find the expected number of games that are played.

(b) If X and Y are independent gamma random variables with parameters (α, λ) and (β, λ), respectively, compute the joint density of $U = X + Y$ and $V = X/(X + Y)$.

Q.4(a) Define Moment Generating Function (MGF). Obtain MGF of normal distribution and hence find its mean and variance.

(b) State and prove Markov's inequality and hence derive Chebyshev's Inequality.

Q.5(a) A prisoner is trapped in a cell containing three doors. The first door leads to a tunnel that returns him to his cell after two days of travel. The second leads to a tunnel that returns him to his cell after three days of travel. The third door leads immediately to freedom. Assume that the prisoner will always select doors 1, 2 and 3 with probability 0.5, 0.3, 0.2, what is the expected number of days until he reaches freedom? Find the variance of the number of days until the prisoner reaches freedom.

(b) If X and Y are independent random variables both uniformly distributed on (0, 1), then calculate the probability density of X + Y.

Q.6(a) Specify the classes of the following Markov chains, and determine whether they are transient or recurrent.

$$P = \begin{Vmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{Vmatrix}$$

(b) For the following transition probability matrix of a three state markov chain, in the long run what proportion of time is the process in each of the three states?

$$P = \begin{Vmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{Vmatrix}$$

Q.7(a) For any random variables X, Y, Z and constant c, prove that

(i) $\text{Cov}(cX, Y) = c\,\text{Cov}(X, Y)$,
(ii) $\text{Cov}(X, Y + Z) = \text{Cov}(X, Y) + \text{Cov}(X,Z)$.

Turn over

(b) Using the principle of Least square, fit a straight line to the following data:

X	5	12	14	17	23	30	40	47
Y	4	10	13	15	15	25	27	46

400