

Sl. No. of Ques. Paper : 6192 F-5
 Unique Paper Code : 2341502
 Name of Paper : Theory of Computation
 Name of Course : B.Tech. Computer Science
 Semester : V
 Duration : 3 hours
 Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is of 35 marks and all its parts are compulsory Attempt any four questions from Q. No. 2 to Q. No. 7.

PART A

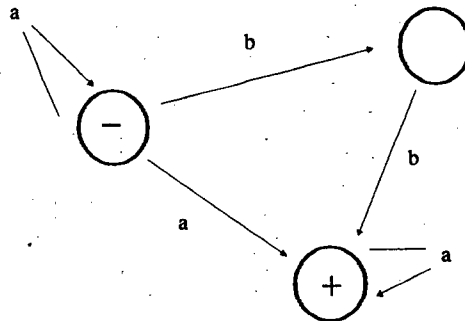
NOTE: For all the questions, consider the alphabet {a, b} unless otherwise specified.

1. (a) For a language defined over the alphabet , is $(a^*b^*)^* = (a+b)^*$? Generate the first 6 words of each of the language in the lexicographic order. 3
- (b) Construct a Finite Automata (FA) for a language having strings that do not end in a double letter, i.e., aa or bb. 3
- (c) Build an FA machine that accepts all strings that have an even length that is not divisible by 6. 3
- (d) Consider the grammar for the language $a^n b^n$:

$$S \rightarrow aS | ab$$

Chomky-ize the grammar. 3

- (e) Convert the following Non-deterministic FA (NFA) to Deterministic FA (DFA): 3



- (f) Find a Context Free Grammar (CFG) for a language of the form $a^x b^y a^z$ where $x, y, z = 1, 2, 3 \dots$ and $x + z = y = \{abba, aabbba, abbbaaa, \dots\}$ 4

- (g) Construct a Push Down Automata (PDA) that accepts strings with unbalanced open and close round braces, where all the opening braces precede the closing braces, *i.e.*, strings of the form $(^n)^m$, where $n, m = 1, 2, 3 \dots$ (*i.e.*, $n, m \in \mathbb{N}$) and $n \neq m$. Some example strings are $\{(), ((), (((), (((), \dots\}$. The alphabet for the language is $\{(), \}$. 4
- (h) Design a Turing Machine (TM) to accept the language with words of the form $a^n b^n a^n$ where $n = 1, 2, 3 \dots$ (*i.e.*, $n, m \in \mathbb{N}$). 4
- (i) Construct a TM that transforms the configuration $Uw\underline{U}$ (where w is an input string with no blanks) into the configuration $UUw\underline{U}$. U is representing the blank symbol and $\underline{\quad}$ shows the current position of the head of the TM. 4
- (j) Use Pumping Lemma to show that the language PALINDROME is non-regular. 4

PART B

2. (a) What language is $\text{PALINDROME} \cap \{a^n b^{n+m} a^m \mid n, m = 1, 2, 3 \dots \text{ (i.e., } n, m \in \mathbb{N})\}$. Is it context free? If context free, draw the PDA, else use Pumping Lemma to show that it is non-CF. 5
- (b) Give a CFG for language with words of type $a^x b^y a^z b^w$, $x, y, z, w = 1, 2, 3 \dots$ $y > x, z > w$ and $x + z = y + w$. 5

3. (a) Consider the CFG in Chomsky Normal Form (CNF):

$$S \rightarrow PQ$$

$$Q \rightarrow QS/b$$

$$P \rightarrow a$$

5

Generate the derivation trees for the words (i) abab, (ii) ababab.

Consider Q as the self embedded non terminal, trace the division of each word w into $uvxyz$ and $uvvxyyz$, where $|u| + |z| \geq 0$, $|v| + |y| > 0$ and $|x| > 0$.

- (b) Consider the following languages:

$$L_1 = \{a^n b^m \text{ where } n \geq m\}$$

$$L_2 = \{a^n b^m \text{ where } m \geq n\}$$

What is the language formed by their intersection? Show that this language is context free by constructing a PDA. 5

4. (a) Use pumping lemma to show that language $\{a^n b^{2n} \mid n = 1, 2, 3 \dots\}$ is non-regular. 4

- (b) For the languages $L_1 = (a+b)^*a$ and $L_2 = (a+b)^*aa(a+b)^*$ construct the respective FAs and derive the finite automata that define $L_1 \cap L_2$. 1+2+3
5. (a) Consider the homomorphism h from the alphabet $\{0, 1, 2\}$ to $\{a, b\}$ defined by:
- $$h(0) = ab, h(1) = b, h(2) = aa.$$
- (i) If L is the language $(ab + baa)^*bab$, what is $h^{-1}(L)$? 4
- (ii) If L is the language consisting of the single string $ababb$, what is $h^{-1}(L)$? 3
- (b) Given the language represented by $(1+0)^*1$, show that the reverse of the language is also regular using a DFA. 3
- (c) Construct a DFA accepting all strings w over $\{0, 1\}$ such that the number of 1's in w is $3 \pmod 4$. 3
6. (a) Give the regular expression for the following language:
- (i) All the strings in which b 's occur in clumps of an odd number at a time such as $abaabbbab, ab, ababbba, \dots$
- (ii) All words that contain exactly two b 's or exactly three b 's. 3+2
- (b) If L is a recursive language, then prove that its complement is also recursive. 5
7. (a) What does the following notation represent:
 $U("M" "w") = "M(w)"$, where $M = (K, \Sigma, \delta, s, H)$ and U is the Universal TM. 5
- (b) Design a Turing Machine for finding the two's complement of a given number which is provided as input to it in binary form over the alphabet $\{0, 1\}$. 5