This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1358**

Unique Paper Code : **2351505** **F-7**

Name of the Paper : **Numerical Methods**

Name of the Course : **B.Tech. (Electronics)**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *Two* parts from each question.

Use of Scientific Calculators are allowed.

1. (*a*) Solve the following equation using bisection method for the smallest positive root :

$$x^3 - 4x + 1 = 0$$

Find the root after three iteration. How do you compare Bisection method with Newton's method of root finding ? 6½

(*b*) Apply Secant method to $x^4 - x - 10 = 0$ to determine an approximation to a root lying in the interval (1, 2) correct to two decimal places. 6½

(c) The negative root of the smallest magnitude of the equation

$$f(x) = x^3 - 3x^2 + 17x - 51 = 0$$

is to be obtained by Newton-Raphson method. 6½

2. (a) Use Newton's method to solve the given Non-Linear system of equations :

$$f(x, y) = x^2 + y^2 - 1 = 0$$

$$g(x, y) = x^2 - y = 0$$

Take Initial Approximation $(x_0, y_0) = (0.5, 0.5)$ and perform three iterations. 6

(b) Find the inverse of the following matrix using Gauss-Jordan elimination method : 6

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 11 \\ 3 & 11 & 8 \end{bmatrix}$$

(c) Use Gauss-Seidel method to solve the given system of equations :

$$2x_1 - x_2 + x_3 = -1$$

$$x_1 + 2x_2 - x_3 = 7$$

$$x_1 - x_2 + 2x_3 = -10$$

Take Initial Approximation $X^0 = (0, 0, 0)^T$ and perform 3 iterations. 6

3. (*a*) The following values of the function $f(x) = e^x$, are given :

x	-1	0	1
$f(x)$	e^{-1}	1.0	e^1

Construct the quadratic interpolating polynomial with the help of Lagrange's method that fits the data. Hence, calculate $f(0.5)$ to find the exact error in the value. 6½

(*b*) Construct divided difference table for the following the data :

x	0.5	1.5	3.0	5.0	6.5
$f(x)$	1.625	5.875	31.0	131.0	282.125

Prove that :

(*i*) $\Delta\left(\frac{1}{f_i}\right) = \frac{\Delta f_i}{f_i f_{i+1}}$

(*ii*) $\Delta f_i + \nabla f_i = \Delta f_i / \nabla f_i - \nabla f_i / \Delta f_i$. 6½

(*c*) For the following data, construct the backward difference table and obtain backward difference polynomial :

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

Interpolate the function at $x = 0.25$. 6½

4. (*a*) Obtain piecewise linear interpolating polynomials for the function $f(x)$ defined by the data :

x	1	2	4	8
$f(x)$	3	7	21	73

Hence estimate the value of $f(3)$ and what can you say about error at $x = 2$. 6

(*b*) The following table of values is given :

x	−1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

using the formula $f'(x_1) = \dfrac{f(x_1 + h) - f(x_1 - h)}{2h}$ and the Richardson extrapolation, find $f'(3)$. 6

(*c*) Evaluate the integral :

$$\int_0^1 \frac{1}{1+x^2}\,dx.$$

using composite Trapezoidal's Rule for $n = 5$. Hence find an approximation to π ? 6

5. (*a*) Derive Simple Trapezoidal Rule with error term. What is the order for the method ? 6

(*b*) Compute :

$$\int_0^2 e^{-x^2}\,dx$$

using Gaussian quadrature by three-point formula. 6

(c) Apply Euler's method to approximate the solution of the Initial Value Problem :

$$\frac{dy}{dx} = -x^2 y, \qquad y(1) = 2$$

over the interval [1, 2] with $h = 0.2$. 6

(a) Given the Initial Value Problem :

$$\frac{dy}{dx} = 1 - 2xy, \qquad y(0) = 1$$

Estimate $y(0.8)$, using Ralston's method, with $h = 0.2$. 6½

(b) Given the Initial Value Problem :

$$\frac{dy}{dx} = xy, \qquad y(0) = 1$$

with $h = 0.5$ on the interval [0, 1] using classical 4th order Runge-Kutta method. 6½

(c) Apply finite-difference method to solve the problem :

$$\frac{d^2y}{dx^2} = x + y, \qquad 0 \leq x \leq 1$$

with $y(0) = 2$, $y(1) = 2.5$ and $h = 0.25$. 6½

ks : 75

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P.T.O.