This question paper contains **4+1** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1331**

Unique Paper Code : **2341501** **F-7**

Name of the Paper : **Probability Theory and Statistical Computing**

Name of the Course : **B.Tech. Computer Science**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory. (7×5=35 marks)

Attempt any *four* questions from **2** to **7**. (2×5=10 marks each)

Parts of a question must be answered together.

Use of Non-Programmable Scientific Calculator is allowed.

The symbols have their usual meaning.

1. (*a*) State Baye's theorem. There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin ?

(*b*) Derive Poisson distribution as a limiting case of Bionomial Distribution.

(*c*) Let X be a random variable with probability density :

$$f(x) = \begin{cases} c\left(1 - x^2\right), & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

(*i*) What is the value of c ?

(*ii*) What is the cumulative distribution function of X ?

(*d*) The joint density of X and Y is :

$$f(x, y) = \begin{cases} \dfrac{\left(y^2 - x^2\right)e^{-y}}{8}, & 0 < y < \infty, -y \le x \le y. \end{cases}$$

Show that $E[X|Y = y] = 0$.

(*e*) If X_1 and X_2 are independent binomial random variables with respective parameters (n_1, p) and (n_2, p), calculate the conditional probability mass function of X_1 given that $X_1 + X_2 = m$.

(*f*) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Transform the above process as Markov chain and write its transition probability matrix.

(g) Data was collected on x = shear force (kgs) and y = percent fiber dry weight to test the toughness and fibrousness of asparagus as a major determinant of quality. The following observations were recorded :

$$n = 16, \sum x_i = 1.656, \sum x_i^2 = 0.196912, \sum y_i = 170.6, \sum y_i^2 = 2253.56, \sum x_i y_i = 20.0397$$

Calculate the value of sample correlation coefficient and also compute the coefficient of determination. Interpret the results.

2. (a) Suppose that each of three men at a party throws his hat into the center of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men selects his own hat ?

(b) The probability of winning on a single toss of the dice is p. A starts, and if he fails, he passes the dice to B, who then attempts to win on her toss. They continue tossing the dice back and forth until one of them wins. What are their respective probabilities of winning ?

3. (a) If X is a non-negative integer valued random variable, show that :

$$E[X] = \sum_{n=1}^{\infty} P\{X \geq n\} = \sum_{n=0}^{\infty} P\{X > n\}.$$

(b) If X and Y are independent gamma random variables with parameters (α, λ) and (β, λ), respectively, compute the joint density of U = X + Y and V = X/Y.

4. (a) Define Moment Generating Function (MGF). Obtain MGF of exponential distribution and hence find its mean and variance.

(b) State and prove Central Limit theorem.

5. (*a*) A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time and variance until the miner reaches safety ?

(*b*) Let X and Y be independent Poisson random variables with respective means λ_1 and λ_2. Calculate the distribution of X + Y.

6. (*a*) Let $\{X_n, n \geq 0\}$ be a Markov Chain having state space S = {1, 2, 3, 4} and transition probability matrix :

$$P = \begin{Vmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{Vmatrix}$$

Prove that the states 1 and 2 are Ergodic.

(*b*) A flea moves around the vertices of a triangle in the following manner. Whenever it is at vertex i it moves to its clockwise neighbor vertex with probability p_i and to the counterclockwise neighbor with probability $q_i = 1 - p_i$, $i = 1, 2, 3$. Find the proportion of time that the flea is at each of the vertices.

7. (*a*) Suppose that X_1, ..., X_n are independent and identically distributed with expected value μ and variance σ^2. Then prove that :

$Cov(\bar{X}, X_i - \bar{X}) = 0$, $i = 1$, ..., n. Where $\bar{X}$ is the sample mean.

(*b*) Using the principle of Least square, fit a straight line to the following data :

X	5	10	11	12	31	37	42	58
Y	48	55	48	50	58	52	80	86