This question paper contains 4 printed pages.

440

Your Roll No. .....

B.A. (Hons.) / II **E**

DISCIPLINE CENTRED CONCURRENT
COURSE – ECONOMICS

[For Economics (Hons.)]

(MATHS : Elements of Analysis)

(Admissions of 2005 and onwards)

Time : 2 hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are 3 Sections. Attempt all the Sections.
Marks are indicated against each question.
Attempt any two questions from each Section.

SECTION I

1. (a) Show that for all real numbers x and y

$$\frac{|x+y|}{1+|x+y|} \leq \frac{|x|}{1+|x|} + \frac{|y|}{1+|y|}.$$ 4

(b) Define supremum and infimum of a set of real numbers. Give an example of a set whose supremum and infimum do not belong to it. 4

P. T. O.

2. (a) Prove that every convergent sequence is bounded but the converse need not be true. 4

(b) Prove that:

$$\lim_{n\to\infty} n^{1/n}=1.$$ 4

3. (a) Show that the sequence $<a_n>$ defined as $a_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\frac{1}{n!}$ is a Cauchy sequence. 4

(b) State Cauchy's First Theorem on limits. Use it to show that the sequence $<a_n>$, where

$$a_n=\frac{1}{n}\left(1+\frac{1}{2}+....+\frac{1}{n}\right)$$

is convergent. 4

Section II

4. (a) Show that:

$$\sum_{n=1}^{\infty}\frac{1}{2n-1}$$

does not converge. 3

(b) Test for convergence of the series:

$$\sum_{n=1}^{\infty}\frac{n^2}{(n^3+4)}$$ 3

5. Test the following series for convergence:

(i) $\sum_{n=1}^{\infty} \frac{x^n}{(n+1)\sqrt{(n+2)}}$, for $x>0$

(ii) $\sum_{n=1}^{\infty} \frac{1.2.3 \ldots n}{7.10 \ldots (3n+4)}$ 3+3

6. Define Alternating series. Test for the convergence and absolute convergence of the following series:

$$\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\ldots$$ 6

Section III

7. Determine the radius of convergence and the interval of convergence of the following power series:

$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2} x^n.$$ 5

8. Show that

$$\tan^{-1} x=x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots\ldots, \quad -1<x<1.$$ 5

9. Show that if a power series $\sum_{n=1}^{\infty} a_n x^n$ converges for

$x=x_0$ then it is absolutely convergent for every $x=x_1$, when $|x_1|<|x_0|$. 5