[This question paper contains 4 printed pages.]

442 *Your Roll No.* ....

B.A. (Hons.) / II **E**

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *question in all, selecting* **two** *questions from each section.*

SECTION I

1\. (a) (i) Define a basis of vector space R^3 over R.

(ii) Check whether the given set

$S = \{(2,2,3),(-1,-2,1),(0,1,0)\}$ spans R^3 or not. (4)

P.T.O.

(b) (i) Determine whether the given subset of R^3 is a subspace or not.

$$W = \{(x_1, x_2, x_3) : 2x_1 + x_2 - x_3 = 0\}$$

(ii) Given a subspace $W = \{(a, b, c) : a - 2b + 3c = 0\}$ of $R^3(R)$. Find its basis. (4)

2. (a) (i) Find T (x, y) for every $(x, y) \in R^2$ if T is a linear transformation such that

$$T(1, 0) = (1, 1),\ T(0, 1) = (-1, 1)$$

(ii) Let $T : R^2 \to R^2$ defined as $T(x_1, x_2) = (1 + x_1, x_2)$. Show that T is not Linear Transformation. (4)

(b) Let $T : R^3 \to R^2$ be defined by $T(x, y, z) = (x + y, y - z)$. Let

$\beta_1 = \{(1, 1, 0), (0, 1, 0), (-1, 1, 1)\}$ and $\beta_2 = \{(-1, 1), (1, 2)\}$. Find $[T]_{\beta_1, \beta_2}$. (4)

3. (a) Let $u, v \in R^n$, then show that

$$\|u + v\|^2 + \|u - v\|^2 = 2(\|u\|^2 + \|v\|^2)$$ (4)

(b) Find the non-trivial solution, if any, by using elementary row transformations

$$3x + 2y + 7z = 0$$
$$4x - 3y - 2z = 0$$
$$5x + 9y + 23z = 0$$ (4)

SECTION II

4. Prove that the following function is continuous at $x = 0$.

$$f(x) = \begin{cases} x\cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \qquad (6)$$

5. State Intermediate value Theorem. Let f be a function defined by :

$$f(x) = \begin{cases} \frac{1}{2} - x, & 0 \leq x \leq \frac{1}{2} \\ \frac{3}{2} - x, & \frac{1}{2} < x < 1 \\ 2, & x = 1 \end{cases} \qquad (6)$$

Does f possesses Intermediate value property ? Is the function f continuous at $x = \frac{1}{2}$?

6. State Lagrange's Mean Value Theorem . Use it to prove the following inequality :

$|\tan^{-1}x - \tan^{-1}y| < |x - y| \quad \forall x, y \in R.$ (6)

SECTION III

7. Show that the function f, where

$$f(x,y)=\begin{cases}\dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y)\neq(0,0)\\ 0 \quad , & \text{if } (x,y)=(0,0)\end{cases}$$

is continuous at (0,0) and not differentiable at (0,0). (5)

8. Show that the function $f(x, y) = (y-x)^4 + (x-2)^4$ has minimum at (2, 2). (5)

9. For each of the limits

$\lim_{x\to 0}\lim_{y\to 0} f(x, y)$, $\lim_{y\to 0}\lim_{x\to 0} f(x, y)$ and $\lim_{(x,y)\to(0,0)} f(x, y)$,

determine it exist or not, where

$$f(x,y)=\frac{x^3y}{2x^6+y^2} \quad \text{whenever } (x, y) \neq (0,0)$$ (5)