This question paper contains 4 printed pages.

441 *Your Roll No.* ....

B.A. (Hons.) / II **E**

DISCIPLINE CENTRED CONCURRENT COURSE

[For Economics (H)]

MATHS : Elements of Analysis

(Admissions of 2005 and onwards)

Time : 2 hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are 3 Sections. Attempt all the Sections.
Marks are indicated against each question.
Attempt any two questions from each Section.

SECTION I

1. (a) State and prove the Archimedean property of real numbers. 4

   (b) Does the sequence $<(-1)^n+3>$ converge? Justify your answer. 4

2. (a) State and prove Squeeze Theorem. 4

   (b) Using Cauchy's Convergence Criterion for sequences show that the sequence $<a_n>$, where $a_n =$

$$1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n},$$ does not converge. 4

3. (a) Define a monotonic sequence. Let:—

$$a_1=\sqrt{5},\ a_{n+1}=\sqrt{5+a_n}\ .$$

Show that $<a_n>$ is monotonic and bounded. Also find its limit. 4

(b) State Cauchy's Second Theorem on limits. Prove that:—

$$\lim_{n\to\infty}\left[\frac{(2n)!}{(n!)^2}\right]^{1/n}=4.$$ 4

SECTION II

4. (a) Show that $\sum\frac{1}{n}$ does not converge. 3

(b) Test for convergence of the series:

$$\sum_{n=1}^{\infty}(\sqrt{n+1}-\sqrt{n}).$$ 3

5. Test the following series for convergence:—

(i) $\sum_{n=1}^{\infty}\frac{\sqrt{n-1}}{\sqrt{n^3+1}}x^n,\quad \text{for } x>0$

(ii) $\sum_{n=1}^{\infty} \frac{1}{n!}$ 3+3

6. Define Alternating Series. Test for the convergence and absolute convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sin(n\alpha)}{n^3}, \text{ where } \alpha \in \mathbf{R}.$$ 1+5

SECTION III

7. Determine the radius of convergence of the following power series:—

(i) $\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

(ii) $\sum_{n=1}^{\infty} \frac{n!}{n^n}(x+2)^n$ 5

8. Show that:

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots, \quad -1 < x < 1.$$ 5

9. Define sine and cosine functions as sums of power series. Prove that for all real numbers x, y:

$$S(x+y)=S(x)C(y)+C(x)S(y)$$

where S and C denote sine and cosine respectively. 5

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