[This question paper contains 4 printed pages.]

443 *Your Roll No.* ....

B.A. (Hons.)/II **E**

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admission of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any ***six*** *question in all, selecting two question from each Section.*

Section-I

1.(a)(i) Show that $W = \{(a_1, a_2, a_3,) : a_3 = a_1 + a_2\}$ is subspace of R^3.

(ii) Prove that intersection of two subspace of R^n is a subspace of R^n. (4)

(b)(i) Check whether the given set S = {(1,1,0), (2,1,1), (3,0,3)} is linearly dependent or independent.

(ii) Give a subspace W = $\{(x_1, x_2, x_3) : x_1 - x_2 - x_3 = 0\}$ of R^3 (R). Find its basis. (4)

2.(a)(i) Show that T : $R^2 \to R^3$ defined as

$T(x_1, x_2) = (x_1, x_1 + x_2, x_2)$ is a linear transformation.

(ii) Let T : $R^2 \to R^2$ be defined by T (x, y) = (x, x + y). Let

β_1 = {(1, 0), (0, 1)} and β_2 = (1, 2), (1, 1)}. find $[T]\beta_1,\beta_2$. (4)

(b) Solve the system of equations

$x + 3y - z = 0$

$2x - y + 4z = 0$

$x - 11y + 14z = 0$

Using row operations. (4)

3.(a)(i) Find angle between u = (1,-2,0,2) and v = (-3,6,0,-6).

(ii) let u and v be orthogonal vectors. Show that

$\| u + v \|^2 = \| u \|^2 + \| v \|^2$ (4)

(b) Define Norm of a vector. Let u = (1,-2,1), v = (3,1-2), w=(-2,1,3) $\in R^3$. Find vector 't' such that <t, u>=1, <t, v> = -1 and <t, w>=6, where <,> is the standard inner product. (4)

Section-II

4. Examine the continuity at x=1 and x=2 of the function f, where f is defined as:

$$f(x)=\begin{cases} 2x & , \quad 0 \le x \le 1 \\ 2-x & , \quad 1 < x \le 2 \\ x^2-2x & , \quad x=2 \end{cases}$$ (6)

5. Let $f(x)=\begin{cases} x\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x=0 \end{cases}$

Show that f is continous but not differentiable at x = 0. (6)

6. State Rolle's Theorem. Examine the validity of the hypothesis and the conclusion of Rolle's Theorem for the function:

$f(x) = 1 - x^{2/3} \quad \forall\ x \in [-1, 1].$ (6)

Section-III

7. Show that the function f, where

$$f(x, y) = \begin{cases} \dfrac{x^2y}{x^4+y^2} & , \text{if } (x, y) \neq (0,0) \\ 0 & , \text{if } (x, y) = (0,0) \end{cases}$$

has directional derivatives in all direction at (0,0) but is not continuous at that point. (5)

8. Find the maxima and minima of the function $f(x, y) = 4x\ y - x^4 - y^4$. (5)

9. For each of the limits

$\lim_{x\to 0} \lim_{y\to 0} f(x,y)$, $\lim_{y\to 0} \lim_{x\to 0} f(x,y)$ and $\lim_{(x,y)\to(0,0)} f(x,y)$, determine it exist or not, where

$$f(x,y) = \frac{(x^2y + xy^2)\sin(x-y)}{x^2+y^2} \text{ whenever } (x,y) \neq (0,0)$$

(5)