This question paper contains 4 printed pages]

Your Roll No. ....

5665

B.A. (Hons.)/II **D**

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all, selecting *two* questions from each Section.

SECTION I

1. (*a*) Define Vector Space $R^3(R)$. Show that :

(*i*) If $a.x = 0$, then $a = 0$ or $x = 0$, where $x \in R^3$ and $a \in R$.

(*ii*) $(-a).x = a.(-x) = -(a.x)$, where $a \in R$ and $x \in R^3$. 4

P.T.O.

(*b*) Define basis of Vector Space $R^3(R)$. Find a basis of the subspace W of R^3, where :

$$W = \{(x, y, z) : 2x + y - z = 0\}.$$ 4

2. (*a*) Define $T : R^2 \to R^2$ as :

$$T(x, y) = (2x, x + y).$$

Show that T is a Linear Transformation. Find $[T]_\beta$, where $\beta = \{(1, 0), (0, 1)\}$. 4

(*b*) Verify Rank Nullity Theorem, for the Linear Transformation $T : R^3 \to R^3$ such that :

$$T(x, y, z) = (x + y, x - y, 0).$$ 4

3. (*a*) State and prove Cauchy-Schwarz inequality. 4

(*b*) Define Hermitian and Unitary Matrices. If U is a unitary matrix ? show that $|\det U| = 1$. 4

SECTION II

4. Use $\varepsilon-\delta$ definition to prove that :

$$\lim_{x\to 2}(x^2 - x + 1) = 3.$$ 6

5. Let

$$f(x) = |x-1| + |x|,\ x \in R.$$

Find the points at which the function is not differentiable. 6

6. State Lagrange's Mean Value Theorem. Show that :

$$|\sin x - \sin y| \le |x-y|\ x, y \in R.$$ 6

SECTION III

7. Let

$$f(x, y) = x\sin 1/x + y\sin 1/y,\ xy \neq 0.$$

Show that :

$$\lim_{x\to 0}\lim_{y\to 0} f(x, y) \text{ and } \lim_{y\to 0}\lim_{x\to 0} f(x, y)$$

do not exist but $\lim f(x, y)$ exists at (0, 0). 5

8. Show that $f(x, y)$ is differentiable at (0, 0), where :

$$f(x, y) = \begin{cases} \dfrac{x^2 y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$ 5

9. Show that the function :

$$f(x, y) = (y - x)^4 + (x - y)^2$$

has minimum at (1, 1). 5