This question paper contains 4 printed pages]

Your Roll No. ....

5663

B.A. (Hons.) – II **D**

DISCIPLINE CENTRED CONCURRENT COURSE

[For Economics (H)]

MATHS : Elements of Analysis

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

Section I

1. (*a*) If T is a non-empty set of real numbers which is bounded above, then a real number t is the supremum of T if and only if the following two conditions hold :

   (*i*) $x \leq t \quad \forall\, x \in T$

   (*ii*) Given any $\varepsilon > 0$, there exists some $t \in T$ such that $x > t - \varepsilon$. 4

(*b*) Define a convergent sequence. Using the definition of convergence, show that if $p > 0$, then $\lim_{n \to \infty} \frac{1}{n^p} = 0$. 4

2. (*a*) If $<a_n>$ and $<b_n>$ are two convergent sequences with $\lim a_n = a$, $\lim b_n = b$, show that $<a_n + b_n>$ is also convergent and

$$\lim (a_n + b_n) = a + b.$$ 4

(*b*) Define Cauchy's first theorem on limits and show that :

$$\lim_{n \to \infty} \frac{1}{n}\left[1 + \frac{1}{2} + \ldots\ldots + \frac{1}{n}\right] = 0.$$ 1+3

3. (*a*) Show that every Cauchy's sequence is bounded but the converse is not true. 4

(*b*) Show that the sequence $<S_n>$ defined by $S_{n+1} = \sqrt{7 + S_n}$, $S_1 = \sqrt{7}$ converges to the positive root of $x^2 - x - 7 = 0$. 4

Section II

4. (*a*) Let Σu_n and Σv_n be two positive term series such that :

$$\lim_{n \to \infty} \frac{u_n}{v_n} = l, \quad (l \text{ is finite and non-zero}).$$

Then show that both the series converge or diverge together.

(*b*) Show that the series $\frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \ldots$ does not converge. 4+2

5. Test for convergence of the series :

(*a*) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots.$ 3

(*b*) $\frac{1^2 2^2}{1!} + \frac{2^2 3^2}{2!} + \frac{3^2 4^2}{3!} + \ldots.$ 3

6. (*a*) Test for convergence the series whose *n*th term is $\frac{n^{n^2}}{(n+1)^{n^2}}$. 3

(*b*) Test for convergence and absolute convergence of the series $\sum \frac{(-1)^{n-1}}{n^2}$. 3

Section III

7. Write down the power series expansion of e^x. 5

8. Define the cosine function $C(x)$ and the sine function $S(x)$. Prove that C and S satisfy Pythagorean identity $(C(x))^2 + (S(x))^2 = 1 \ \forall \ x \in R$. 5

9. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{n^n} x^n$. 5