This question paper contains 4 printed pages]

Your Roll No. ....

5662

B.A. (Hons.) – II **D**

Discipline Centred Concurrent Course–Economics

(For Economics Hons.)

(Maths : Elements of Analysis)

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* questions from each Section.

Section I

1. (*a*) If x and y are two real numbers then show that :

$$|x - y| \geq ||x| - |y||.$$ 4

(*b*) Show that the sequence $<r^n>$ converges to zero if $|r| < 1$. 4

2. (*a*) If $<a_n>$ and $<b_n>$ are two convergent sequences with $\lim a_n = a$, $\lim b_n = b$, show that $<a_n b_n>$ is also convergent and

$$\lim (a_n b_n) = ab.$$ 4

(*b*) Define Cauchy's first theorem on limits and show that :

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}\ldots+\frac{1}{\sqrt{n^2+n}}\right]=1.$$ 1+3

3. (*a*) State Cauchy's General Principle of convergence and use it to prove that the sequence defined by

$$a^n = 1+\frac{1}{3}+\frac{1}{5}+\ldots.+\frac{1}{2n-1}$$

does not converge. 1+3

(*b*) Prove that the sequence $<a_n>$ defined by the relation

$$a_n = 1,\quad a_n = 1+\frac{1}{1!}+\frac{1}{2!}+\ldots..+\frac{1}{(n-1)!}\ (n \geq 2),$$

converges. 4

Section II

4. Prove that the necessary condition for a series Σu_n to converge is $\lim_{n \to \infty} u_n = 0$. Show by an example that the converse is not true. 3+3

5. (*a*) Test for convergence of the series

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$$ 3

(*b*) Test for convergence of the series whose *n*th term is

$$\frac{n^{n^2}}{(n+1)^{n^2}}.$$ 3

6. Show that the following series converges :

$$\frac{\log 2}{2^2} - \frac{\log 3}{3^2} + \frac{\log 4}{4^2} - \ldots$$ 6

Section III

7. Write down the power series expansion of $\sin x$. 5

8. Define logarithm function L. State at least *two* properties of logarithm function. 5

9. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n}$ 5