This question paper contains 4 printed pages]

Your Roll No........................

5664

B.A. (Hons.)/II **D**

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admissions of 2005 and onwards)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Six* questions in all selecting *two* questions from each Section.

Section I

1. (*a*) Prove that intersection of two subspaces of $R^3(R)$ is also a subspace of R^3. Give an example to show that union of two subspace may not be a subspace. 4

(*b*) Find a basis of the following subspaces of the vector space R^3 over R :

(*i*) $W_1 = \{(x, o, z) : x, z \in R\}$

(*ii*) $W_2 = \{(x, y, z) : x = y - z\}$. 4

2. (*a*) Let $T : R^3 \to R^3$ be defined by

$$T(x, y, z) = (x + y, x, y - z).$$

Find matrix of T w.r.t. standard ordered basis β, hence find $T(1, -1, 0)$. 4

(*b*) Verify Rank-Nullity Theorem for the Linear Transformation $T : R^3 \to R^3$ such that $T(x, y, z) = (0, y - z, x + y)$. 4

3. (*a*) If u and v are vectors in R^3 then prove that

$$\|u + v\| \leq \|u\| + \|v\|.$$

Verify the inequality for $u = (1, -1, 0)$ and $v = (1, 2, 3)$. 4

(*b*) Define an orthonormal basis of R^3 over R.

Prove that :

$$S = \left\{\left(\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right), \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right), \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)\right\}$$

forms an orthonormal set in R^3 w.r.t. standard inner product. 4

Section II

4. If

$$\lim_{x \to c} f(x) = L$$

then show that :

$$\lim_{x \to c} |f(x)| = |L|.$$

Give an example of a function f such that $\lim_{x \to c} |f(x)|$ exists but $\lim f(x)$ does not exist. 6

5. Let

$$f(x) = \begin{cases} x^2 & x \le c \\ ax + b & x > c \end{cases}$$

where a, b and c are constants.

Find the values of a and b in terms of c such that $f'(c)$ exists. 6

6. State Rolle's theorem and hence show that there is no real number b for which the equation $x^2 - 3x + b = 0$ has two distinct roots in [0, 1]. 6

Section III

7. Let

$$f(x, y) = \frac{x^3 y}{2x^6 + y^2} \quad (x, y) \neq (0,0).$$

Determine the two repeated limits $\lim_{x \to 0} \lim_{y \to 0} f(x, y)$ and $\lim_{y \to 0} \lim_{x \to 0} f(x, y)$. Also determine whether $\lim f(x, y)$ exists or not at (0, 0). 5

8. Let

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

Show that f possesses partial derivatives at (0, 0) but is not differentiable at (0, 0). 5

9. Find the maxima and minima of the function :

$$f(x, y) = 4xy - x^4 - y^4$$ 5