[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8140 **D** **Your Roll No................**

Unique Paper Code : 235483

Name of the Course : **B.A. (H) Discipline Centered Concurrent Course (other than Economics)**

Name of the Paper : Algebra and Calculus

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. Attempt **six** more questions selecting any **two** questions from each section.

1. (a) If $y = \log(5x^2 + 7)$, find $\frac{dy}{dx}$. (3×5)

(b) Evaluate $\lim\limits_{x\to\infty} \frac{2x^2 - 5x - 3}{3x^2 - x - 20}$.

(c) Let $A = \begin{bmatrix} 3 & -4 \\ -5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 4 \\ 5 & x \end{bmatrix}$, what values of x, if any will make AB = BA.

(d) Evaluate $\int \frac{x^2}{x^3 + 5} dx$.

(e) Let $A = \begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix}$, find A^{-1}.

SECTION A

1. (a) If $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$, k is any positive integers. (5)

(b) Solve the system of linear equations with the help of Cramer's Rule :

$x + y + z = 7$

$x + 2y + 3z = 16$

$x + 3y + 4z = 22$ (5)

2. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Also verify that $AA^{-1} = A^{-1}A = I_2$ where A^{-1} denote the inverse of the matrix A. (5)

(b) Show that the value of the determinant $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$. (5)

3. (a) Find the equation of the straight line parallel $2x + 3y + 11 = 0$ and which is such that the sum of its intercepts on the axis is 15. (5)

(b) Show that the equation $ax^2 + ay^2 + 2gx + 2fy + c = 0$, $a \neq 0$ represent a circle, also find radius and centre of the circle. (5)

SECTION B

4. (a) If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$. (5)

(b) Show that the function f(x) defined by

$$f(x) = \begin{cases} 5x - 4, & \text{if } 0 < x \le 1 \\ 4x^3 - 3x, & \text{if } 1 < x \le 2 \end{cases}$$

is continuous at x = 1. (5)

5. (a) Examine the concavity of the function :

$f(x) = x^3 - 3x^2 + 3x - 3$ (5)

(b) Find all the points of local maxima and local minima as well as the corresponding local maximum and minimum values of the function.

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 1$$ (5)

6. (a) Verify Rolle's theorem for

$f(x) = x^3 - 6x^2 + 11x - 6$ in [1,3] (5)

(b) Obtain Maclaurins' series expansion of

$f(x) = e^x$ (5)

SECTION C

7. (a) Evaluate $\int_0^\infty \frac{x^3}{(1+x^2)^{\frac{9}{2}}} dx$. (5)

(b) Find the length of the arc of the curve $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$ from x = 0 to x = 3. (5)

8. (a) In a certain culture of bacteria, the number of bacteria increased six fold in 10 hours. How long did it take for the population to double ? (5)

(b) Find the area of the region enclosed by the curve $y = x^2$, and $y^2 = x$. (5)

9. (a) Suppose that in the course of any given year, the number of cases of a diseases is reduced by 20%. If there are 10,000 cases today, how many years will it take to reduce the number to 1000. (5)

(b) Evaluate : $\int \frac{1}{1 - \sin x + \cos x} dx$. (5)