Instructions for candidates
Q1 is compulsory.
Attempt any 5 questions out of the remaining 6 questions (Q2-Q7)
Use of non-programmable calculators is allowed.

Q1.(a) The prices per ton of wheat and rye are p_1 and p_2 respectively. The market demand for wheat is given by $x_1 = 4 - 10p_1 + 7p_2$ and for rye by $x_2 = 3 + 7p_1 - 5p_2$. The supply of wheat is related by the relation $x_1 = 7 + p_1 - p_2$ and the supply of rye by $x_2 = -27 - p_1 + 2p_2$.

(i) Write down the relevant simultaneous equations, for determining the equilibrium prices, in matrix form.

(ii) Find equilibrium prices and quantities.

Q1.(b) Approximate the function $y = (3x+4)/(x-2)$, by a linear function when x is approximately equal to 4.

Q1.(c) The arithmetic mean and standard deviation of series of 20 items were calculated by a student as 20 cms and 5 cms respectively. But while calculating them an item 13 was misread as 30. Find the correct arithmetic mean and standard deviation.

Q1.(d) Find Karl Pearson's coefficient of correlation between Wages and Cost of living and comment.

Wages	100	01	103	102	104	99	97	98	96
Cost of living	98	99	99	97	95	92	95	94	90

Q1.(e) A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years ? Use a 0.05 level of significance.

[5X5=25]

Q2. (a)Show that the vectors $x_1 = (1,2,4)$, $x_2 = (2,-1,3)$, $x_3 = (0,1,2)$ and $x_4 = (-3,7,2)$ are linearly dependent.

Q2.(b) The cost of producing x units is given as :

$C(x) = 0.001x^3 - 0.3x^2 + 30x + 42$

1

Determine where the cost function is concave up and where it is concave down. Also find the inflexion point.

OR

Find the optimum value of x and y for the function $z = 10x + 20y - x^2 - y^2$ under the condition that $2x + 5y = 10$

[5X2=10]

Q3.(a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then find the value of $x(\partial u/\partial x) + y(\partial u/\partial y) + z(\partial u/\partial z)$

Q3.(b) The total revenue function is given by :

$$R(x) = \begin{cases} \frac{(10x^2 - 160)}{(x-4)} & \text{when } x \neq 4 \\ k & \text{when } x = 4 \end{cases}$$

Find the value of the constant k, if revenue function is continuous at x=4.   [5x2=10]

Q4.(a) In a bolt factory, machines A,B and C manufacture respectively 25%, 35% and 40% of the total production of their output. A, B and C manufacture 5%,4% and 2% defective bolts out of their respective outputs. A bolt is drawn at random and found to be defective. What is the probability that it was manufactured by machine A.

Q4.(b) Let X be a random variable with probability density

$$f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1 \\ 0 & otherwise \end{cases}$$

(a) What is the value of c ?
(b) $P[(1/2) < x < (3/4)] = ?$

OR

The joint probability distribution of a pair of random variables is given by the following table:

X→ Y↓	1	2	3
1	1/6	1/12	0
2	0	1/9	1/5
3	1/18	1/4	2/15

Evaluate

i)   The marginal probability of X=1
ii)   Conditional probability mass function of Y given X = 2   [5X2=10]

Q5.(a) A miner is trapped in a mine containing 3 doors. The first door leads to a tunnel that takes him to safety after 2 hours of travel. The second door leads to a tunnel that returns him to the mine after 3 hours of travel. The third door leads to a tunnel that returns him to the

2

mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, what is the expected length of time until the miner reaches safety.

Q5.(b) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes 200 bottles. A drug manufacturer buys 500 boxes from the producer of bottles. Find how many boxes will contain at least two defectives.

[5X2=10]

Q6.(a)Let X be the number of times that a fair coin, flipped 40 times, lands heads. Find the probability that X=20. Use the normal approximation and the compare it to exact solution.

Q6.(b) Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500. What can be said about the probability that this week's production will be atleast 1000. Comment using Markov inequality.   [5X2=10]

Q7.(a) The Edison Electric Institute has published figures on the annual number of kilowatt hours expended by various home appliances. It is claimed that a vacuum cleaner expends an average of 46 kilowatt-hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners expend an average of 42 kilowatt-hours per year with a standard deviation of 11.9 kilowatt-hour, does this suggest at the 0.05 level of significance that vacuum cleaners expend on an average less 46 kilowatt-hour annually. Assume the population of kilowatt-hours to be normal.

Q7.(b)State the Central Limit Theorem.
The lifetime of a special type of battery is random variable with mean 40 hours and standard deviation 20 hours. A battery is used until it fails, at which point it is replaced by a new one. Assuming a stockpile of 25 such batteries the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained.

OR

Explain what is meant by Type I and Type II errors in Testing of Hypothesis.   [5x2=10]