

This question paper contains 4+1 printed pages]

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S. No. of Question Paper : 7847

Unique Paper Code : 2351102

F-1

Name of the Paper : Algebra—I [DC-1.2]

Name of the Course : Bachelor with Honours

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All six questions are compulsory.

Do any two parts from each question.

1. (a) Find the sixth roots of the complex number $(1 + i)$ and represent them in the complex plane. 5

(b) Solve the equation : 5

$$z^{10} + (-2 + i)z^5 - 2i = 0.$$

(c) (i) Express $\sin 6\theta$ in powers of $\sin \theta$ and $\cos \theta$. 3

(ii) Find the geometric image of the complex number z , where : 2

$$|z + i| \geq 2.$$

2. (a) Let \sim denotes an equivalence relation on set A . Let $a \in A$ then prove that for any $x \in A$, $x \sim a$ if and only if $\bar{x} = \bar{a}$. 5

P.T.O.

- (b) For $a, b \in \mathbb{Z}$, define $a \sim b$ if and only if $a^2 - b^2$ is divisible by 3. 5
- (i) Prove that \sim defines an equivalence relation on \mathbb{Z} .
- (ii) What is $\bar{0}$ and $\bar{1}$?
- (c) Prove that the intervals $(2, 5)$ and $(10, \infty)$ have the same cardinality. 5
3. (a) If $a = bq + r$ for integers a, b, q and r then prove that $\text{g.c.d.}(a, b) = \text{g.c.d.}(b, r)$. 5
- (b) (i) If $ac \equiv bc \pmod{n}$ and $\text{g.c.d.}(c, n) = 1$ then prove that $a \equiv b \pmod{n}$. 3
- (ii) Find x such that $(1080)^5 \equiv x \pmod{7}$. 2
- (c) Use the principle of Mathematical induction to prove that : -5

$$n! > n^3 \quad \forall n \geq 6.$$

4. (a) Consider the following system of linear equations :

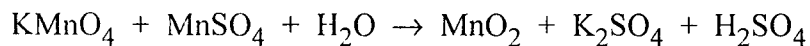
$$x_1 + x_2 - 2x_3 + 4x_4 = 5$$

$$2x_1 + 2x_2 - 3x_3 + x_4 = 3$$

$$3x_1 + 3x_2 - 4x_3 - 2x_4 = 1$$

- (i) Write the matrix equation and the vector equation of the above system of equations.
- (ii) Find the general solution in parametric vector form by reducing it into Echelon form.
- (iii) List the pivot columns. 7.5

- (b) Balance the given chemical equation where the reaction between potassium permanganate (KMnO_4) and manganese sulphate in water produces manganese dioxide, potassium sulphate and sulphuric acid :



Here for each compound, construct a vector that lists the numbers of atoms of potassium (K), manganese (Mn), oxygen (O), sulfur (S) and hydrogen (H).

Find h and k such that it has unique solution, infinite solutions and no solution. 5,2.5

- (c) (i) Define linear independence of n vectors. How many pivot columns must a 7×5 matrix have if its columns are linearly independent ? Why ?

(ii) If :

$$u_1 = (1, -3, 2), u_2 = (2, -4, -1),$$

$$u_3 = (1, -5, 7) \text{ and } u_4 = (2, -5, 3)$$

belongs to \mathbb{R}^3 .

Check whether $u_4 \in \{u_1, u_2, u_3\}$ or not. Is $\{u_1, u_2, u_3, u_4\}$

linearly dependent or linearly independent ? 5,2.5

5. (a) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined as :

$$T(x, y, z) = (x - z, -2y + z, 3x - y)$$

- (i) Prove that T is a linear transformation.
- (ii) Find the standard matrix of T .
- (iii) Let $u = (-2, 3, 1)$, find $T(u)$ using the standard matrix of T . 3,3,1.5
- (b) (i) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation. Then prove that T is one to one if and only if the equation $T(x) = 0$ has only the trivial solution.
- (ii) Show that the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ given by

$$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 + x_3)$$

is one to one and onto.

3.5,4

- (c) (i) Define projection mapping from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ an x -axis and y -axis.
- (ii) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be linear transformation that first reflects points through the horizontal axis and then reflects points through the line $y = x$. Find the standard matrix of T . 3,4.5

6. (a) Find the basis and dimension for Column space of A and Null space of A

where :

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix}$$

Hence find rank of A.

7.5

(b) Define subspace of \mathbb{R}^n . Let $H = \{(a, b, c) \in \mathbb{R}^3 \mid c = 2a + b\}$

(i) Show that H is a subspace of \mathbb{R}^3 .

(ii) Prove that the Eigen values of a triangular matrix are the entries on the main diagonal.

4,3.5

(c) Let :

$$A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

(i) Find Eigen values of A.

(ii) Find Eigen vectors and Eigen space corresponding to each Eigen value.

3.5,4