This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7847**

Unique Paper Code : **2351102** **F-1**

Name of the Paper : **Algebra—I [DC-1.2]**

Name of the Course : **Bachelor with Honours**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory.

Do any *two* parts from each question.

1. (*a*) Find the sixth roots of the complex number $(1 + i)$ and represent them in the complex plane. 5

(*b*) Solve the equation : 5

$$z^{10} + (-2 + i)z^5 - 2i = 0.$$

(*c*) (*i*) Express sin 6θ in powers of sin θ and cos θ. 3

(*ii*) Find the geometric image of the complex number z, where : 2

$$|z + i| \geq 2.$$

2. (*a*) Let ~ denotes an equivalence relation on set A. Let $a \in$ A then prove that for any $x \in$ A, $x \sim a$ if and only if $\bar{x} = \bar{a}$. 5

P.T.O.

(*b*) For $a, b \in Z$, define $a \sim b$ if and only if $a^2 - b^2$ is divisible by 3. 5

(*i*) Prove that $\sim$ defines an equivalence relation on Z.

(*ii*) What is $\bar{0}$ and $\bar{1}$?

(*c*) Prove that the intervals (2, 5) and $(10, \infty)$ have the same cardinality. 5

3. (*a*) If $a = bq + r$ for integers a, b, q and r then prove that g.c.d.(a, b) = g.c.d.(b, r). 5

(*b*) (*i*) If $ac \equiv bc(\text{mod } n)$ and g.c.d.$(c, n) = 1$ then prove that $a \equiv b(\text{mod } n)$. 3

(*ii*) Find x such that $(1080)^5 \equiv x(\text{mod } 7)$. 2

(*c*) Use the principle of Mathematical induction to prove that : 5

$$n! > n^3 \ \forall \ n \geq 6.$$

4. (*a*) Consider the following system of linear equations :

$$x_1 + x_2 - 2x_3 + 4x_4 = 5$$

$$2x_1 + 2x_2 - 3x_3 + x_4 = 3$$

$$3x_1 + 3x_2 - 4x_3 - 2x_4 = 1$$

(*i*) Write the matrix equation and the vector equation of the above system of equations.

(*ii*) Find the general solution in parametric vector form by reducing it into Echelon form.

(*iii*) List the pivot columns. 7.5

(*b*) Balance the given chemical equation where the reaction between potassium permanganate ($KMnO_4$) and manganese sulphate in water produces manganese dioxide, potassium sulphate and sulphuric acid :

$$KMnO_4 + MnSO_4 + H_2O \rightarrow MnO_2 + K_2SO_4 + H_2SO_4$$

Here for each compound, construct a vector that lists the numbers of atoms of potassium (K), manganese (Mn), oxygen (O), sulfur (S) and hydrogen (H).

Find h and k such that it has unique solution, infinite solutions and no solution. 5,2.5

(*c*) (*i*) Define linear independence of n vectors. How many pivot columns must a 7×5 matrix have if its columns are linearly independent ? Why ?

(*ii*) If :

$$u_1 = (1, -3, 2), u_2 = (2, -4, -1),$$

$$u_3 = (1, -5, 7) \text{ and } u_4 = (2, -5, 3)$$

belongs to R^3.

Check whether $u_4 \in \{u_1, u_2, u_3\}$ or not. Is $\{u_1, u_2, u_3, u_4\}$ linearly dependent or lineary independent ? 5,2.5

5. (*a*) Let $T : R^3 \to R^3$ be defined as :

$$T(x, y, z) = (x - z, -2y + z, 3x - y)$$

(*i*) Prove that T is a linear transformation.

(*ii*) Find the standard matrix of T.

(*iii*) Let $u = (-2, 3, 1)$, find $T(u)$ using the standard matrix of T. 3,3,1.5

(*b*) (*i*) Let $T : R^n \to R^m$ be a linear transformation. Then prove that T is one to one if and only if the equation $T(x) = 0$ has only the trivial solution.

(*ii*) Show that the linear transformation $T : R^3 \to R^3$ given by

$$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 + x_3)$$

is one to one and onto. 3.5,4

(*c*) (*i*) Define projection mapping from $R^2 \to R^2$ an *x*-axis and *y*-axis.

(*ii*) Let $T : R^2 \to R^2$ be linear transformation that first reflects points through the horizontal axis and then reflects points through the line $y = x$. Find the standard matrix of T. 3,4.5

6. (*a*) Find the basis and dimension for Column space of A and Null space of A where :

$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & 5 & 5 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix}.$$

Hence find rank of A. 7.5

(*b*) Define subspace of R^n. Let $H = \{(a, b, c) \in R^3 \mid c = 2a + b\}$

(*i*) Show that H is a subspace of R^3.

(*ii*) Prove that the Eigen values of a triangular matrix are the entries on the main diagonal. 4,3.5

(*c*) Let :

$$A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 1 & 6 \\ 0 & 0 & 2 \end{bmatrix}.$$

(*i*) Find Eigen values of A.

(*ii*) Find Eigen vectors and Eigen space corresponding to each Eigen value. 3.5,4