This question paper contains 3 printed pages.]

Your Roll No. ....

5233

B.A. (Hons.) Programme B

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admission of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all, selecting **two** questions from each section.

SECTION - I

1. (a) Prove that intersection of two subspaces of R^n is a subspace of R^n. Show that union of two subspaces of R^n need not be a subspace of R^n. 4

(b) Define linearly independent subset of a vector space R^3 over R. 4

Determine whether the following subset

S of R^3 is linearly independent or not

$S = \{2, 1, 4), (-3, -2, -1), (1, -3, -2)\}$

2. (a) Describe $T: R^3 \rightarrow R^3$ such that

$$[T]_B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

Where B is the standard basis of R^3 4

(b) Define a projection on the Xu plane. Obtain its matrix w.r.t. the standard bases of R^3 and R^2. 4

3. (a) State and prove Cauchy Schwary inequality. 4

(b) If $S = \{u_1, u_2, u_3\}$ is an orthogonal set of nonzero vectors in R^3, show that S is linearly independent. 4

Does it constitute a basis of R^3 ?

SECTION-II

4. Use $\in - \delta$ definition to prove that the function.

$f(x) = x^2, x \in [-3, 2]$ 6

is continuous at $x = 1$

5. Let function f be defined as follows :

$$f(x) = \begin{cases} x^2 + 3x + a & \text{if } x \leq 1 \\ bx + 2 & \text{if } x > 1 \end{cases}$$ 6

Find the values of a and b such that f is derivable at $x = 1$

6. State Lagrange's Mean Valve theorem. Use it to prove that 6

$$|\tan^{-1} x - \tan^{-1} y| \leq |x - y| \ \forall x, y \in R.$$

SECTION-III

7. Show that for the function f given by

$$f(x,y) = \begin{cases} x \operatorname{Sin}\frac{1}{y} + y \operatorname{Sin}\frac{1}{x}, & \text{when } xy \neq 0 \\ O, & \text{when } xy = 0 \end{cases}$$ 5

none of the two repeated limits exists, but $\lim_{(x,y)\to(0,0)} f(x, y)$ exists.

8. Show that $f(x,y) = y^4 + x^2y + x^4$ has minimum at (0,0). 5

9. Prove that the function

$$f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ O, & (x,y) = (0,0) \end{cases}$$ 5

is continuous, possesses partial derivatives but not differentiable at (0,0).
