[This question paper contains 4 printed pages.]

5232 *Your Roll No.* ....

B.A. (Hons.) Programme **B**

DISCIPLINE CENTRED CONCURRENT COURSE

[For Economics (H)]

MATHS : Elements of Analysis

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *questions from each Section.*

SECTION A

1. (a) Find the Supremum and infimum of the following sets if they exist

 (i) $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$

 (ii) $\left\{(-1)^n : n \in \mathbb{N}\right\}$ (4)

 (b) Define convergent sequence. Use this definition to show that

$$\left\langle \frac{n-1}{n} \right\rangle \to 1$$ (1+3)

P.T.O.

2. State Cauchy's Convergence Criterion. Hence show that the sequence $<a_n>$ defined as :

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \text{----} + \frac{1}{n}$$

does not converge. (2+6)

3. Define $<a_n>$ as :

$$a_1 = 1, \quad a_{n+1} = \sqrt{2 + a_n}$$

Show that $<a_n>$ is monotonic and bounded. Also find its limit. (6+2)

SECTION B

4. Test the following Series for convergence or divergence :

(i) $\frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 3^2} + \frac{1}{3 \cdot 3^3} + \frac{1}{4 \cdot 3^4} + \text{----}$

(ii) $\sum_{n=1}^{\infty} \frac{\sqrt{n-1}}{\sqrt{n^3+1}} x^n, \; x > 0$ (3+3)

5. Test the Convergence of the following Series :

(i) $\sum_{n=1}^{\infty} \left(\sqrt{n^3+1} - \sqrt{n^3}\right)$

(ii) $\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \text{----}$ (3+3)

6. Use the alternating Series test to determine the convergence of the following Series :

(i) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n+3}{n(n+1)}$

(ii) $\sum_{n=1}^{\infty} (-1)^{n+1} e^{-n}$ (3+3)

SECTION C

7. Define cosine and sine functions as sums of power series. Prove that

(i) $S(x+y) = S(x)\,C(y) + C(x)\,S(y)$

(ii) $C(x+y) = C(x)\,C(y) - S(x)\,S(y)$

Where C and S denote cosine and sine respectively. (5)

8. Show that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \quad -1 < x \leq 1$$

and deduce that

$$\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$$ (5)

9. Determine the radius of convergence and interval of convergence of the following series

$$\sum_{n=1}^{\infty} \frac{(n!)^2 x^{2n}}{(2n)!}$$ (5)