This question paper contains 3 printed pages.]

Your Roll No. .....

5228

B.A. (Hons.) Programme B

DISCIPLINE CENTRED CONCURRENT COURSE

(Maths for other than Economics)

(Algebra and Calculus)

(Admission of 2005 and onwards)

Time : 2 Hours **Maximum Marks : 38**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory and carries **eight** marks.

Attempt **six** more questions from the remaining Question No. **2** to **10**, selecting **two** questions each from Sections I, II and III.

Each question carries **5** marks.

1. (i) Let $f(x) = \begin{cases} 3-x & , \ x < 2 \\ \frac{x}{2}+1 & , \ x > 2 \end{cases}$

Does $\lim_{x \to 2} f(x)$ exist ? If so, what is it, if not why not ? 2

(ii) Does the set of vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right\}$ span $\mathbb{R}^2$? Justify. 2

(iii) Find $\frac{dy}{dx}$ for the function $y = \cos^2(3x^3 + y)$. 2

(iv) Find the length of the curve $y = x^2$, $-1 \leq x \leq 2$. 2

SECTION – I

2. Solve the following system of linear equations :

$$x + y + 3z = 1$$
$$2x + 3y - z = 3$$
$$5x + 7y + z = 7$$

3. Find the equation of the parabola that is symmetric about the y-axis, has its vertex at the origin and passes through the point (5, 2).

4. Let $f(x) = \frac{x^3}{4} + 1$ on [0, 2]. Use Mean Value Theorem to show that at some point C in (0, 2), the tangent is inclined at an angle of $\frac{\pi}{4}$ to the x-axis. Find this C by actual calculation.

SECTION – II

5. Let $f(x) = \ln x = \log_e x$, $\frac{1}{3} \leq x \leq 3$. Find points x where the slope of the tangent to the above curve is (i) 1, (ii) $\frac{1}{2}$, (iii) 2.

Display this in a rough sketch.

What is $f(x)$ when $x = e^2$?

6. Find the greatest and the least values of the function $f(x) = x^3 - 9x^2 + 24x$ in $[0, 6]$.

7. Write McLaurin's series for the function $f(x) = \cos x$ for any real no. x.

SECTION – III

8. Evaluate

(i) $\int \cos^3 2x \, dx$

(ii) $\int x(x^2 + 1)^{2/3} dx$

9. Find the arc length of the curve $y = 3x^{\frac{3}{2}} - 1$ from $x = 0$ to $x = 1$.

10. The population of a country is assumed to grow as $y(t) = y(0)\, e^{kt}$ where t is the time elapsed from the beginning, y(t) is the population at time t and k is the growth constant.

If the population doubles itself every 40 years, find k.

Also express the growth rate in percentage.
