[This question paper contains 4 printed pages.]

5234 *Your Roll No.* ....

B.A. (Hons.) Programme **B**

DISCIPLINE CENTRED CONCURRENT COURSE

ECONOMICS

(For Economics Hons.)

(Maths : Linear Algebra and Calculus)

(Admissions of 2005 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **six** *questions in all, selecting* **two** *questions from each section.*

SECTION I

1. (a) Define a subspace of vector space R^3 over R. Determine whether the following subsets of R^3 are subspaces or not :

(i) $W_1 = \{(x, y, z) / x + y = z\}$

(ii) $W_2 = \{(x, y, z) / x = 3z, y = 2x\}$ (4)

(b) Find the linear span of subset S of vector space R^3 over R, where

(i) $S = \{(x, 2y, x + 2y) : x, y \in R\}$

(ii) $S = \{(x, y, z) : x = 0\}$ (4)

2. (a) Find a matrix of linear transformation

$T : R^3 \rightarrow R^3$ defined by

$T(x, y, z) = (x + y + z, -y, y + 2z)$

w.r.t. standard bases. (4)

(b) Verify rank-nullity theorem for linear transformation

$T : R^2 \rightarrow R^2$ such that

$T(x, y) = (x, x + y, y)$ (4)

3. (a) If u and v are vectors in R^n, then prove that

$$|\langle u, v \rangle| \leq \|u\| \, \|v\|$$

where $|\cdot|$ on the left hand side stands for absolute value of real number and $\langle , \rangle$ is a standard inner product on R^n. (4)

(b) For an orthogonal matrix A, show that $\det A = \pm 1$.

Give example of a 2×2 matrix A, such that A is orthogonal and $\det A = -1$. (4)

SECTION II

4. (a) Use $\epsilon - \delta$ definition to prove that the following function is continuous at $x = 0$

$$f(x) = \begin{cases} x \cos\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \qquad (6)$$

5. (a) State Intermediate Value theorem.

(b) Let f and g be continuous on [a, b] and $f(a) < g(a)$ but $f(b) > g(b)$. Prove that $f(c) = g(c)$ for some c in (a, b). (2+4)

6. State Lagrange's Mean theorem.

Show that the function

$$f(x) = x^{1/3}, \quad x \in [-1, 1]$$

does not satisfy the hypothesis of Langrange Mean Value theorem but still satisfies the conclusion.

(2+4)

SECTION III

7. Show that if $f(x, y)$ is differentiable at a point (a, b) in the domain of f, then f is continuous at (a, b). Show that the converse is not true. (5)

8. Show that the following function

$$f(x, y) = x^2 - 3xy^2 + 2y^4$$

has neither a maximum nor a minimum value at the origin. (5)

9. State Taylor's theorem for a function of two variables and use it to expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) upto terms of second degree. (5)